Interaction in Prevention: A General Theory and an Application to COVID-19 Pandemic

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Abstract

Prevention efforts often involve spillovers, positive or negative, on other individuals, but this is neglected by standard models of risk prevention. We analyze strategic interaction between decision makers whose effort affects each other's risk. We characterize response functions and Nash equilibria, providing proof of existence and analyzing the Pareto efficiency and possible multiplicity of equilibria. We then analyze the optimal effort level from a social point of view, finding conditions under which Nash equilibria are characterized by under– or over–provision of effort, which calls for policy interventions. Finally, we specialize our model to describe the risk of COVID–19 infection. The features of contagion are consistent with the existence of asymmetric equilibria where the high effort exerted by one decision maker pushes another to exert low effort. Moreover, socially optimal mandatory policies, for instance concerning face masks, can cause a decision maker to decrease exerted effort.

JEL Codes: D81, C72, I12
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1 Introduction

The risk of incurring bad events is frequent in the lives of individual decision makers. These events may involve financial damage, for example patrimonial loss or income reduction, but they also include health and physical problems, like disease or accident. A typical way to deal with this kind of risk is trying to reduce the probability that the bad event will occur through actions known as "risk prevention".

Prevention has been widely studied in decision theory literature. Starting with the seminal paper by Ehrlich and Becker (1972), it was discussed with relation to the choice of a decision maker who faces the risk of incurring a future loss, of any potential type, and has the option to reduce the probability that the loss will occur by exerting some kind of costly effort. Many issues have been analyzed in the literature. Dionne and Eeckhoudt (1985), Briys and Schlesinger (1990), and Eeckhoudt and Gollier (2005) examine the role of preferences in determining optimal choices. Sweeney and Beard (1992) consider the effect of a change in the size of the possible loss. Jindapon and Neilson (2007) study the implications of the introduction of random wealth in both states of nature. Menegatti (2009) analyses prevention in a multi-period context. Eeckhoudt et al. (2012) and Courbage and Rey (2012) examine the effects of the introduction of a background risk. Chuang et al. (2013) and Crainich et al. (2016) consider the effects of changes in risk of different orders.[1] Crainich and Menegatti (2021) analyse the implications of random costs of prevention.

So prevention in general has been widely studied, but, to our knowledge, the fact that preventive actions exhibit interactions has been almost entirely ignored. In fact, when an individual tries to prevent a bad event and reduce the probability that this event will occur, he often also affects the probability that a similar event will occur for someone else. A good example of this was seen in the recent COVID-19 pandemic, where it is very clear that when one person takes protective measures, such as wearing a mask or washing hands, he reduces not only his own probability of contagion but also the probability of contagion for other people. Similarly, in a different context, if a car driver avoids drinking before driving and drives carefully, he will reduce not only his own probability of a car accident but also the probability of an accident for other road users. These are two examples where, while trying to prevent a risk for himself, a decision maker also generates a *reduction* in the probability that the bad event will occur to other people. The opposite effect is also however

[1] For a general recent classification of effects of high-order changes in risk see also Menegatti and Peter (2021).

possible. In fact, in some cases, if one agent prevents something for himself, he will *increase* the probability that another agent incurs a bad event. For instance, installing a burglar alarm and putting iron bars on windows will reduce the probability of burglary in one's own home, but it will increase the probability that the burglar targets a neighbor's house. Similarly, if a defendant hires a good lawyer to represent him in a trial this may lower his chances of being convicted but it may also increase the probability that the other party loses.

These effects, which are important issues in prevention, are not however studied in the literature. [2] The first aim of the present paper is to fill this gap in the literature by studying the implications of interaction in preventive actions made by two different decision makers in the usual Ehrlich and Becker (1972) prevention model. We study strategic decisions in this framework and find the optimal choice for each decision maker, given the choice made by the other. On the basis of this reaction function, we determine the decentralized Nash equilibria by considering the situations where both decision makers make their optimal responses.

This analysis distinguishes two circumstances. The first is the case where the two efforts in prevention made by the two decision makers reinforce each other. This is the case where the efforts of one decision maker reduce the probability of a bad event for the other, as described above in the case of COVID–19 prevention measures and careful driving. The second case is that where the two efforts conflict, as in the case where the effort of one decision maker increases the probability that the other incurs a bad event, as described above for burglar alarms. Our results show that this distinction is important in determining the features of the different equilibria which may arise, but that the distinction is not the only relevant element. In fact, our analysis highlights that both marginal benefits and marginal costs of prevention are affected by interactions and that the total effect depends on the way in which different partial effects are combined. In this direction, a characterization of different possible sets of equilibria is provided.

The presence of possible interactions in prevention activities opens the space for a comparison between choices made by individuals in a decentralized

[2] A partial exception is a recent note by Salanié and Treich (2020), who study a model of infection prevention in a pandemic where the probability of contagion of one agent depends on the prevention effort exerted by other agents. However, they consider a continuum of agents with specific parametrized utilities, focusing on a policy maker's introduction of compulsory effort in prevention and disregarding strategic interaction. Accordingly, their model deviates from the standard model of prevention by Ehrlich and Becker (1972) (examined in the present work), and exhibits a specific formalization, based on Hoy and Polborn (2015).

decision process and choices that may be optimal for the society as a whole. The idea that individual behavior in prevention can lead the economy to an undesirable equilibrium is associated with the presence of inefficiency, in that the individual agent does not internalize the spillover effects that he generates on other agents into his decision process.

This issue is particularly significant since it implies that, in some cases, individual choice may generate situations of "under-prevention" or "over-prevention". A clear example of under-prevention can be seen in the recent COVID-19 pandemic where, in many cases, individual behavior in terms of prevention has been found to be too weak from a socially optimal standpoint (Toxvaerd, 2020; Heck et al., 2021; Farboodi et al., 2021).

To our knowledge, the issue of prevention being sub-optimal from a social perspective has not to date been covered in the literature.[3] The second aim of this paper is to fill this second gap in the literature by comparing decentralized equilibria with a centralized economy where a planner chooses optimal levels of prevention for all agents. Our analysis shows that sub-optimality may arise in the decentralized framework and provides evidence on what distinguishes cases of under-prevention from cases of over-prevention.

From a mathematical point of view, our model share some elements with two different strands of literature. The case where efforts of the two decision makers conflict exhibits some similarities with rent-seeking models, pioneered by Tullock (1980) and studied under risk aversion by Konrad and Schlesinger (1997).[4] Some important differences are however present. Rent seeking models are designed to describe strategic behavior in a contest and thus assume perfectly negatively correlated outcomes (one agent succeeds if the other fails). On the contrary, the ex-post levels of wealth of the agents in our model are independent, conditional on individual probabilities of loss. Moreover, because of the kind of problem examined, the analysis of rent seeking models typically disregards the comparison between centralized and decentralized choices, which is prominent in our study. On the other hand, the case where the efforts of the two decision makers reinforce each other exhibits some mathematical similarities with models of production in teams

[3]Two exceptions should be noted, although the approaches are completely different from that proposed in the present paper. The first is the paper by Salanié and Treich (2020) mentioned above. The second is Menegatti (2021b), who does not analyze interactions between decision makers and takes the possible sub–optimality of prevention choices (and particularly under–prevention) as a fact and, given this premise, studies whether and how it could be reduced by means of subsidies.

[4]The literature on the rent-seeking model under risk aversion has evolved then in different directions (Cornes and Hartley, 2003; Treich, 2010; Liu et al., 2018; Menegatti, 2021a).

(Holmstrom, 1982; Rasmusen, 1987), but once again, with the difference that in the latter models outcomes are perfectly correlated (all team members win or lose together).

As noted above, the recent COVID-19 pandemic is a very good example of a situation where there is interaction between agent choices on prevention.[5] Starting from this premise, the paper provides a simple application of our general analysis to the use of face masks in preventing contagion. The results provide a possible explanation, based on strategic interaction, for evidence which emerged in many countries that effort levels in prevention during the pandemic varied significantly between people in terms of individual prevention. Moreover, our analysis also provides a new theoretical foundation, based on the comparison between decentralized and centralized equilibria, for the measures taken in many countries to push people to increase effort in preventing infection.

The paper proceeds as follows. Section 2 examines the framework and studies individual choices. Section 3 analyzes the equilibria determined by interactions. Section 4 compares these equilibria with those of a centralized economy. Section 5 considers the application to face masks during a pandemic. Lastly, Section 6 concludes.

2 Individual choices

Consider two Decision Makers, Decision Maker A (DM A) and Decision Maker B (DM B) whose preferences are represented respectively by the utility functions $U(x)$ and $V(x)$, defined over $\mathbb{R}^+$. The two functions exhibit non-satiation ($\frac{\partial U}{\partial x} = U'(x) > 0$ and $\frac{\partial V}{\partial x} = V'(x) > 0$), and risk aversion ($\frac{\partial^2 U}{\partial x^2} = U''(x) < 0$ and $\frac{\partial^2 V}{\partial x^2} = V''(x) < 0$). DM A has an initial wealth equal to W_A and faces the risk of incurring a loss L_A with probability p (i.e. his wealth remains W_A with probability $1-p$ and becomes $W_A - L_A$ with probability p). Similarly, DM B has an initial wealth equal to W_B and faces the risk of incurring a loss L_B with probability q (i.e. his wealth remains W_B with probability $1-q$ and becomes $W_B - L_B$ with probability q). We also assume that each DM can exert a costly effort to reduce the probability of incurring the loss. This implies that p is a decreasing function of the effort e_A and q is a decreasing function of the effort e_B. For given levels of effort, the losses of the two DMs are uncorrelated events. Consistently with the literature, we also assume that the marginal effect of effort decreases when

[5] In general, the prevention model has been applied to health problems (e.g. Courbage and Rey, 2006; Menegatti, 2014). On the specific issue of vaccination decisions, see Nuscheler and Roeder (2016) and Crainich et al. (2019).

the level of effort increases. Since p and q are decreasing functions of e_A and e_B, respectively, this requires that they are both convex. This assumption is usual in prevention models.

This framework is the traditional Ehrlich and Becker (1972) model of prevention with the only exception that we consider two DMs instead of one. We now introduce the new element studied in this work: interaction. We thus assume that p depends not only by e_A but also on e_B and that q depends not only by e_B but also on e_A. This means that the effort in prevention of one DM affects the probability of occurrence of the bad event for the other.

Hence in our framework $p = p(e_A, e_B)$ and $q = q(e_A, e_B)$. The traditional assumptions on the effect of his own effort on DM's probability of loss occurrence made above imply that $p_A(e_A, e_B) = \frac{\partial p}{\partial e_A} < 0$, $q_B(e_A, e_B) = \frac{\partial q}{\partial e_B} < 0$, $p_{AA}(e_A, e_B) = \frac{\partial^2 p}{\partial e_A^2} > 0$ and $q_{BB}(e_A, e_B) = \frac{\partial^2 q}{\partial e_B^2} > 0$. Our new assumptions on interaction in prevention also imply that $\frac{\partial p}{\partial e_B} = p_B(e_A, e_B)$ and $\frac{\partial q}{\partial e_A} = q_a(e_A, e_B)$ are not null. We distinguish two cases. In the first case, the effort of DM A reduces the probability that the other DM faces the bad event. This means that both efforts act in the same direction. We label this case "reinforcing efforts". It is described by the assumptions $p_B(e_A, e_B) < 0$ and $q_A(e_B, e_A) < 0$. The second case is where the effort of one agent increases the probability that the other agent faces the bad event. We label this case "conflicting efforts". It is described by the assumptions $p_B(e_A, e_B) > 0$ and $q_A(e_B, e_A) > 0$.

In this context, the DM A chooses effort e_A in order to solve the following maximization problem:

$$max_{e_A} \quad \mathcal{U}(e_A, e_B) = p(e_A, e_B)U(W_A - L_A - e_A) + [1 - p(e_A, e_B)]U(W_A - e_A). \tag{1}$$

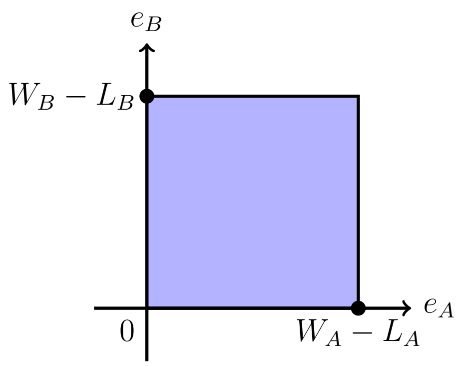
while DM B maximizes the symmetrically defined expected utility $\mathcal{V}(e_A, e_B)$:

$$max_{e_B} \quad \mathcal{V}(e_A, e_B) = q(e_B, e_A)V(W_B - L_B - e_B) + [1 - q(e_B, e_A)]V(W_B - e_B). \tag{2}$$

The action space is constrained by the condition that effort must be weakly positive ($e_A \geq 0$, $e_B \geq 0$) and that wealth must also be weakly positive, including in the case of loss, so $e_A \leq W_A - L_A$ and $e_B \leq W_B - L_B$.[6] We assume that for such levels of effort, p and q only take admissible values

[6] Assuming that the wealth constraint only applies *before* the realization of the loss, i.e. replacing these conditions with $e_A \leq W_A$ and $e_B \leq W_B$ makes no difference for the analysis.

Figure 1: Action space



(in the $[0,1]$ interval).[7] The resulting combined action space for the two DMs is a rectangle, as represented in Figure 1.

We start our analysis by focusing on *internal* solutions to the maximization problem. The first-order condition (FOC) for Problem (1) (DM A) is:

$$\begin{aligned} M_A(e_A, e_B) =& \frac{\partial \mathcal{U}(e_A, e_B)}{\partial e_A} \\ =& p_A(e_A, e_B)[U(W_A - L_A - e_A) - U(W_A - e_A)] \\ & - p(e_A, e_B)U'(W_A - L_A - e_A) - [1 - p(e_A, e_B)]U'(W_A - e_A) = 0 \end{aligned} \tag{3}$$

and for Problem (2) (DM B) it is

$$\begin{aligned} M_B(e_A, e_B) =& q_B(e_B, e_A)[V(W_B - L_B - e_B) - V(W_B - e_B)] \\ & - q(e_B, e_B)V'(W_B - L_B - e_B) - [1 - q(e_B, e_A)]V'(W_B - e_B) = 0 \end{aligned} \tag{4}$$

We assume that the second–order conditions for Problems (1) and (2) are always satisfied. This assumption is introduced to ensure that a DM's best response is unique. Notice that Jullien et al. (1999) show that a sufficient condition for second–order condition (for DM A) to hold is $p_{AA}(e_A, e_B)p(e_A, e_B) \geq 2(p_A(e_A, e_B))^2$. A similar sufficient condition exists for DM B.

[7] Allowing for U and V to take $\mathbb{R}$ as its domain would not affect our analysis. Indeed, it is easy to see that given the assumptions on the utility functions, $\lim_{e_A \to +\infty} \mathcal{U}(e_A, e_B) = \lim_{e_B \to +\infty} \mathcal{V}(e_A, e_B) = -\infty$, while exerting zero effort guarantees a finite utility: the problem of selecting the optimal effort level is bounded from above anyway by the rationality of DMs.

Condition (3) has a clear and simple interpretation: it requires equality between marginal benefit of prevention ($p_A(e_A, e_B)[U(W_A - L_A - e_A) - U(W_A - e_A)]$) and marginal cost of prevention ($p(e_A, e_B)U'(W_A - L_A - e_A) - [1 - p(e_A, e_B)]U'(W_A - e_A)$). Intuitively, it appears that three constrasting effects are at play in the maximizand of Equation (1). On the one hand, an increase in effort (i) increases the probability of the loss not happening, which is an eventuality characterized by relatively large utility and (because of the concavity of U) relatively small marginal utility, and at the same time (ii) decreases p_A, because of decreasing marginal effectiveness of effort. On the other hand, (iii) an increase in effort decreases total wealth, and hence increases the marginal utility of effort, *conditional on p*.

Clearly, both terms in Equation (3) depend on the effort of both DMs, so the optimal choice of e_A depends on the choice made by DM B on e_B, and vice–versa. The key element for determining DM A's optimal reaction to different levels of e_B (i.e., DM A best response curve) is the sign of $\frac{\partial \mathcal{M}_A}{\partial e_B}$. More specifically, DM A's response curve is increasing when $\frac{\partial \mathcal{M}_A}{\partial e_B} > 0$ and decreasing when $\frac{\partial \mathcal{M}_A}{\partial e_B} < 0$. This can be shown in different ways. On one hand, applying the implicit Function Theorem we immediately obtain that $\frac{de_A}{de_B} = -\frac{\frac{\partial M_A}{\partial e_B}}{\frac{\partial M_A}{\partial e_A}}$, implying in turn what is stated above. On the other hand, the same conclusion is obtained from the fact that, if $\frac{\partial \mathcal{M}_A}{\partial e_B} > 0$,the function $\mathcal{U}$ is supermodular while, if $\frac{\partial \mathcal{M}_A}{\partial e_B} < 0$, the function $-\mathcal{U}$ is supermodular.

We now have that:

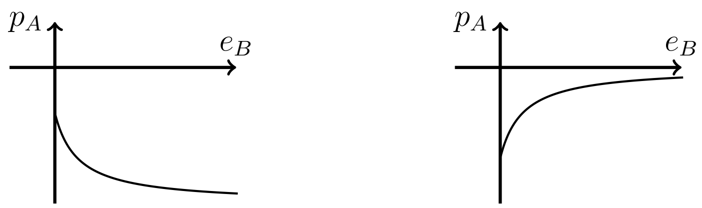
$$\begin{aligned}\frac{\partial M_A}{\partial e_B} =& p_{AB}(e_A, e_B)[U(W_A - L_A - e_A) - U(W_A - e_A)] \\ &+ p_B(e_A, e_B)[U'(W_A - e_A) - U'(W_A - L_A - e_A)] \qquad (5)\end{aligned}$$

where $p_{AB}(e_A, e_B) = \frac{\partial^2 p_A}{\partial e_A \partial e_B}$.

The sign of $\frac{\partial M_A}{\partial e_B}$ depends on the signs of both p_B and of p_{AB}. As already noted, the sign of p_B determines the effect of one DM's effort on the probability that the other DM incurs a bad event. It thus discriminates between the case where the two efforts reinforce each other ($p_B < 0$) and where they are conflicting ($p_B > 0$).

The sign of $p_{AB}(e_A, e_B)$ is difficult to determine *a priori*. Indeed, the cross–derivative measures the marginal effect of a DM's effort on the marginal effectiveness of the other DM's effort, and it can thus be either positive, null, or negative. Note that when $p_{AB} < 0$ the effort of DM B increases the marginal effect of DM A effort while when $p_{AB} > 0$ the effort of DM B reduces the marginal effect of DM A effort. This occurs since p_A always has negative values. As shown in Figure 2a, this means that when $p_{AB} < 0$, an

Figure 2: Possible shape of p_A



(a) $p_A < 0$, $p_{AB} < 0$ **(b)** $p_A < 0$, $p_{AB} > 0$

increase in e_B causes the function p_A to decrease, which implies in turn that e_A has a stronger negative marginal effect on p. The opposite occurs when $p_{AB} > 0$ (Figure 2b).

Lastly, note that p_B and p_{AB} have specific effects on the elements of (3), since the sign of p_B determines how e_B affects the marginal cost of A's own prevention, while the sign of p_{AB} determines how e_B affects the marginal benefit of A's own prevention.

Considering the possible signs of p_B and of p_{AB} we have four cases:

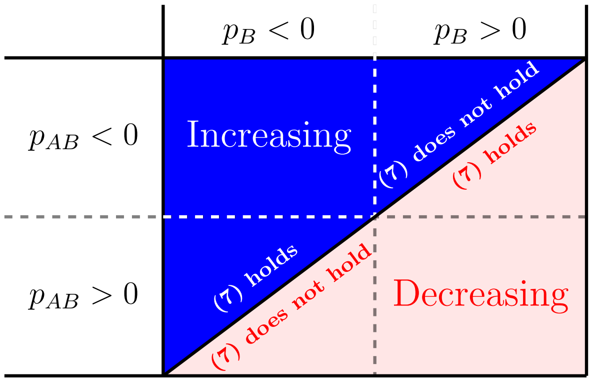
a) $p_B < 0$ and $p_{AB} < 0$
In this case an increase in e_B increases the marginal benefit of e_A and reduces the marginal cost of e_A. The two changes affect choices in the same direction, generating an incentive to increase e_A. Analytically $\frac{\partial M_A}{\partial e_B} > 0$, implying that the reaction curve is increasing ($\mathcal{U}$ is supermodular).

b) $p_B > 0$ and $p_{AB} > 0$
In this case an increase in e_B reduces the marginal benefit of e_A and increases the marginal cost of e_A. The two changes affect choices in the same direction, generating an incentive to reduce e_A. Analytically $\frac{\partial M_A}{\partial e_B} < 0$, implying that the reaction curve is decreasing ($-\mathcal{U}$ is supermodular).

c) $p_B < 0$ and $p_{AB} > 0$
In this case an increase in e_B increases the marginal benefit of e_A and increases the marginal cost of e_A. The two changes affect choices in opposite directions. The shape of the reaction curve depends on which effect prevails. Proposition 1 below shows this case in detail.

d) $p_B > 0$ and $p_{AB} < 0$
In this case an increase in e_B reduces the marginal benefit of e_A and reduces the marginal cost of e_A. The two changes affect choices in

Table 1: Summary of the slope of the reaction function



opposite directions, The shape of the reaction curve depends on which effect prevails. This is also shown in Proposition 1.

Given these cases, a general result can be established:

Proposition 1. *a) When $p_B < 0$, the response curve $e_A^*(e_B)$ is increasing if*

$$e_A \frac{p_{AB}}{p_B} > e_A \frac{U'(W_A - L_A - e_A) - U'(W_A - e_A)}{U(W_A - L_A - e_A) - U(W_A - e_A)} \tag{6}$$

and decreasing when the reversed inequality holds.
b) Conversely, when $p_B > 0$, the response curve $e_A^*(e_B)$ is decreasing if Condition (6) holds and increasing when the reversed inequality holds.

Proof. The proof is trivial since inequality (6) directly comes from (5). □

This proposition is summarized in Table 1. In order to provide a possible interpretation for Condition (6) we consider that $e_A \frac{p_{AB}}{p_B}$ is the elasticity of p_B with respect to e_A and that $e_A \frac{U'(W_A-L_A-e_A)-U'(W_A-e_A)}{U(W_A-L_A-e_A)-U(W_A-e_A)}$ is the elasticity of the utility loss of being in the bad state of nature with respect to e_A.[8] Thus Condition (6) is satisfied if, when e_A changes, the elasticity of the probability of occurrence of loss for DM B is greater than the elasticity of the utility loss of DM A.

We provide a possible interpretation for this condition when $p_B > 0$ and Condition (6) holds. Similar interpretations hold in the other cases. An

[8]Notice that this elasticity is always negative.

increase in e_B raises the probability that a bad event for DM A occurs. Assume now that DM A increases e_A in response to the increase in e_B. This implies that the utility loss decreases in absolute value because of risk aversion (i.e. $U''(.) < 0$). On the other hand, the marginal effect of e_B on p_B can be reduced by the increase in e_A. If this second effect is stronger than the first one, then the optimal response envisages an increase in e_A.

In cases c) and d) above, Condition (6) removes the ambiguity between the effects on marginal benefit and marginal cost characterizing the two cases. Moreover Proposition 1 allows us to identify large classes of problems in which response curves are decreasing or increasing.

While the truth value of Condition (6) depends on the values of e_A and e_B, an interesting aspect is that the right term is independent of e_B. Hence, depending on how the left term changes in e_B we can characterize the space of points (e_A, e_B) where e_A is increasing in e_B. The ratio in the right term of Condition (6) is closely related to the coefficient of absolute risk aversion for U. Specifically, if we consider a constant absolute risk aversion utility $U(x) = 1 - e^{-\beta x}$, the ratio takes value $-\beta$, with β the Arrow-Pratt coefficient of absolute risk aversion. This assumption bears the technical advantage that the initial level of income can be disregarded from the analysis. If instead we consider DMs to exhibit *decreasing* absolute risk aversion, we obtain that the absolute value of the ratio is increasing in effort. In particular, it tends to $+\infty$ asymptotically for a constant relative risk aversion.

The reasoning followed for DM A also implies that DM B has a reaction function where the optimal level of e_B he chooses depends on the value of e_A. The shape of this reaction function depends on a condition similar to (6) that is:

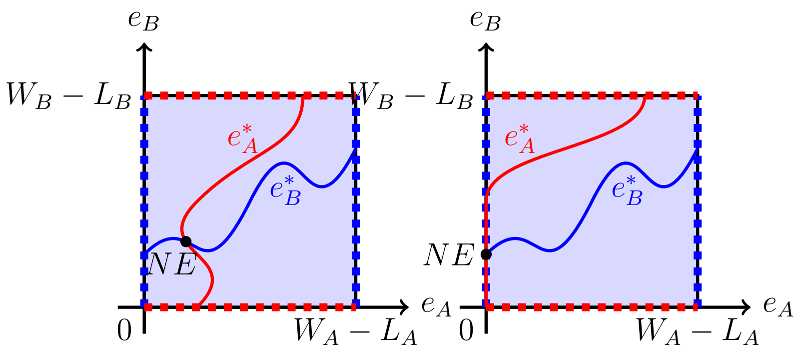
$$e_B \frac{q_{AB}}{q_A} > e_B \frac{U'(W_B - L_B - e_B) - U'(W_B - e_B)}{U(W_B - L_B - e_B) - U(W_B - e_B)}. \tag{7}$$

Conclusions similar to those made above for the reaction function of DM A hold for DM B.

It is worth noting that when both Conditions (6) and (7) are satisfied the game described by functions $\mathcal{U}$ and $\mathcal{V}$ is a supermodular game (Topkis, 1979). Moreover, when Conditions (6) and (7) and efforts are reinforcing, the game is a supermodular game with positive externalities (Milgrom and Roberts, 1990, p. 1267). In a similar way when Conditions (6) and (7) and efforts are conflicting the game is a kind of supermodular game with negative externalities. We will refer to this classification below when commenting our results.

We conclude the analysis of reaction functions with the following general observation which will be useful for later results.

Figure 3: Existence of Nash equilibrium



(a) Proof of Proposition 3: internal Nash equilibrium

(b) Proof of Proposition 3: boundary Nash equilibrium

Lemma 2. *Response functions $e_A^*(e_B)$ and $e_B^*(e_A)$ are continuous.*

Proof. See Appendix A. □

It is worth noticing that this lemma applies both to the internal and boundary components of which a response curve might be the conjunction. Hence, this result allows us to extend Proposition 1 to the entire action space. If for instance e_A^* is increasing in the interior of the action space, by continuity it is also weakly increasing on the entire action space, including its boundaries.

3 Equilibria

Given individual choices of DMs A and B studied in Section 2, we can now easily derive the equilibria of the model in the case of decentralized decisions. Equilibria can be studied graphically by drawing the two reaction functions of DMs A and B in the same Cartesian diagram where e_B and e_A are put on the two axes. As shown in Figure 4 and Figure 5 respectively, either a single equilibrium or multiple equilibria can occur. Nash equilibria can lie either in the interior of the action space, consistently with the analysis of the FOC in Section 2, or on the boundaries. However, at least one Nash equilibrium is certain to exist, as shown in the following proposition.

Proposition 3. *At least one Nash equilibrium necessarily exists.*

Proof. The proof is provided in Appendix A. □

Figure 4: Unique Nash equilibria with increasing and decreasing response curves

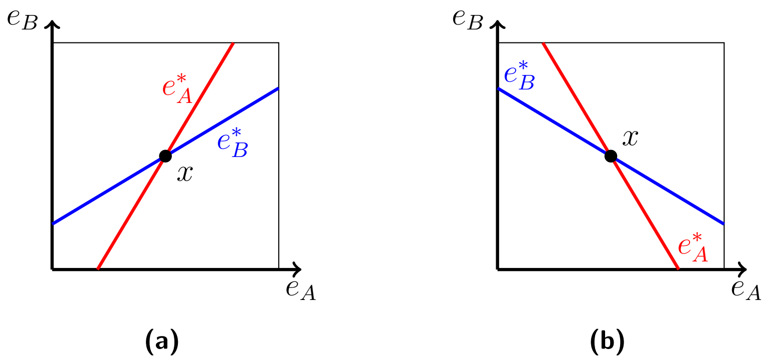
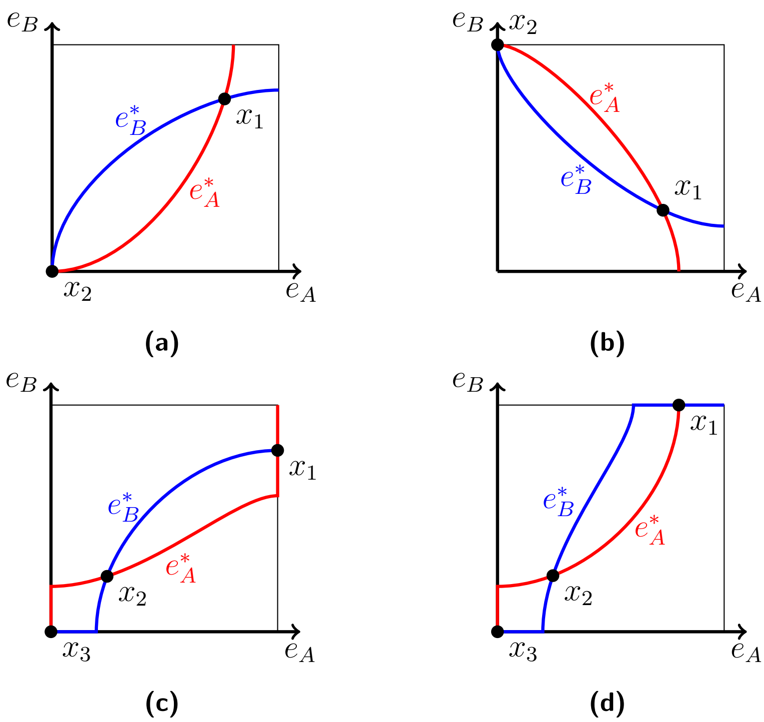


Figure 5: Examples of multiple Nash equilibria



The intuition behind this result is simple: since response functions are continuous, their graphs are continuous curves. The graph of DM A's response curve connects the $e_B = 0$ side and the opposite $e_B = W_B - L_B$ side of the action space, while the graph of DM B's response curve connects the other two sides. So they need to intersect at some point: this point is a Nash equilibrium. Figures 3a and 3b illustrate Proposition 3 in the case of an internal and a boundary Nash equilibrium, respectively.

We now examine the case where there are multiple equilibria. In this case, we have the following results:

Proposition 4. *Moving from an equilibrium to another one, both e_A and e_B change in the same direction (i.e., they either both increase or they both decrease) if reaction curves are increasing (See Proposition 1), while they change in opposite directions (i.e. one increases and the other decreases) if reaction curves are decreasing.*

Proof. Consider the case of increasing reaction curves. Assume without loss of generality that x_1 and x_2 are two equilibria such that A increases effort from x_1 to x_2, while B decreases effort. This would require one of the two reaction curves to be decreasing in an interval between the two equilibrium levels of effort, which contradicts the assumption. The case of decreasing reaction curves is demonstrated similarly. □

Figures 5a and 5b illustrate respectively the two cases of Proposition 4. A special case of Proposition 4 is obtained under the assumption of Case a) presented in Section 2 (see also Table 1), where we have:

Corollary 1. *If $p_B < 0$, $p_{AB} < 0$, $q_A < 0$ and $q_{AB} < 0$, moving from one equilibrium to another the levels of e_A and e_B both change in the same direction. If $p_B > 0$, $p_{AB} > 0$, $q_A > 0$, $q_{AB} > 0$, moving from one equilibrium to another, the levels of e_A and e_B change in opposite directions.*

Two comparative statics results can be derived in the case of reinforcing and conflicting efforts, respectively.

Proposition 5. *In the case of reinforcing efforts ($p_B < 0$, $q_A < 0$), if reaction curves are increasing (see Proposition 1), all Nash equilibria are Pareto ranked and Pareto efficiency increases with DMs efforts; if the reaction functions are decreasing, no Nash equilibria Pareto dominates any other.*

Proof. The proof is provided in Appendix A. □

Proposition 6. *In the case of conflicting efforts ($p_B > 0$, $q_A > 0$), if reaction curves are increasing (see Proposition 1), all Nash equilibria are Pareto* *ranked and Pareto efficiency decreases with DMs efforts; if the reaction functions are decreasing, no Nash equilibria Pareto dominates any other.*

Proof. Similar to the proof of Proposition 5 □

Propositions 5 and 6 have an interesting interpretation. In case of reinforcing efforts, both DMs prefer the equilibrium where efforts are largest while, in case of conflicting efforts, they both prefer the equilibrium where efforts are smallest. But, since there is no coordination, the DMs cannot surely reach this preferred equilibrium and it is possible that a different equilibrium emerges. This suggests that a kind of "under-prevention" may arise in case of reinforcing efforts and a kind of "over-prevention" may arise when efforts are conflicting. A similar conclusion, although in a different sense, is obtained in the next section when comparing a decentralized economy with a centralized economy.

The following result completes our analysis of Pareto efficiency of Nash equilibria.

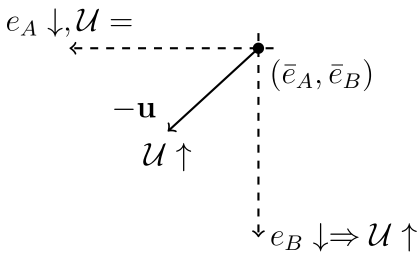
Proposition 7. *In the case of conflicting efforts ($p_B > 0$, $q_A > 0$), any Nash equilibrium in the interior of the action space is Pareto inefficient.*

Proof. Assume that $(\bar{e}_A, \bar{e}_B) \in \mathbb{R}^2_{>0}$ is a Nash Equilibrium located in the interior of the action space. By definition, $\frac{\partial \mathcal{U}}{\partial e_A} = 0$; moreover, $\frac{\partial \mathcal{U}}{\partial e_B} < 0$ (this holds everywhere since $p_B > 0$). Hence, given any vector $\mathbf{u} \in \mathbb{R}^2_{>0}$, the directional derivative $\nabla_{\mathbf{u}} \mathcal{U}(\bar{e}_A, \bar{e}_B)$, which is a linear combination of the two partial derivatives with strictly positive weights $\mathbf{u}_1$ and $\mathbf{u}_2$, is strictly negative. That is, $\mathcal{U}$ increases when moving from $(\bar{e}_A, \bar{e}_B)$ in direction $-\mathbf{u}$ (Figure 6). The same reasoning, applied to DM B, shows that $\mathcal{V}$ increases when moving in direction $-\mathbf{u}$. Hence, in this direction *both* players marginally increase their payoffs, and the proof is concluded. □

Proposition 7 again suggests that contexts with conflicting efforts can result in Pareto–inefficient individual choices, since a coordinated choice with less effort for both agents could increase both agent's utility — a conclusion resembling that of Proposition 6. The difference is that Proposition 7 provides a result which is more general (it requires no monotonicity assumptions on reaction curves) but only applies locally (in a neighborhood of a Nash equilibrium).

It is worth noting that some of our results can be related to the classification of games mentioned in Section 2. In particular, the results in Propositions 3 and 4 and Corollary 1 are coherent with known results on supermodular games (Topkis, 1979) and the results in Proposition 5 are coherent with the conclusions by Milgrom and Roberts (1990) on supermodular

Figure 6: Illustration of Proposition 7 from the point of view of DM A



games with positive externalities. Lastly results in Proposition 6 are parallel conclusions in the case of a supermodular game with negative externalities.

We conclude the analysis of Nash equilibria with a result covering a relatively specific case, which however will be useful in the following section.

Lemma 8. *If response curves are increasing and* $(\bar{e}_A, \bar{e}_B)$ *is a Nash equilibrium such that*

$$\frac{\partial e_A^*(\bar{e}_B)}{\partial e_B} \cdot \frac{\partial e_B^*(\bar{e}_A)}{\partial e_A} > 1 \tag{8}$$

then there is another Nash equilibrium $(\bar{e}'_A, \bar{e}'_B)$ *with higher level of effort for both DMs and such that Equation* (8) *does not hold.*

Proof. See Appendix A. □

Figures 5a, 5c and 5d illustrate this result: given that, in x_2, e_B has a steeper slope than the inverse of e_A, then the Nash equilibrium x_1 must necessarily exist.

4 Centralized economy

This section discusses the optimal choice of e_A and e_B to be made by a centralized planner who chooses the levels of both efforts in order to maximize total utility of the two DMs. The maximization problem of the planner is thus

$$\begin{aligned}\max_{e_A, e_B} \mathcal{C}(e_A, e_B) =& p(e_A, e_B)U(W_A - L_A - e_A) + [1 - p(e_A, e_B)]U(W_A - e_A) + \\ & q(e_B, e_A)V(W_B - L_B - e_B) + [1 - q(e_B, e_A)]V(W_B - e_B)\end{aligned} \tag{9}$$

We first focus on internal solutions. The FOCs with respect to e_A and e_B are respectively:

$$\begin{aligned}\mathcal{C}_A(e_A, e_B) =& p_A(e_A, e_B)[U(W_A - L_A - e_A) - U(W_A - e_A)] \\ & - p(e_A, e_B)U'(W_A - L_A - e_A) - [1 - p(e_A, e_B)]U'(W_A - e_A) \\ & + q_A(e_A, e_B)[V(W_B - L_B - e_B) - V(W_B - e_B)] = 0 \qquad (10)\end{aligned}$$

and

$$\begin{aligned}\mathcal{C}_B(e_A, e_B) =& q_B(e_B, e_A)[V(W_B - L_B - e_B) - V(W_B - e_B)] \\ & - q(e_B, e_B)V'(W_B - L_B - e_B) - [1 - q(e_B, e_A)]V'(W_B - e_B) \\ & + p_B(e_A, e_B)[U(W_A - L_A - e_A) - U(W_A - e_A)] = 0 \qquad (11)\end{aligned}$$

In the same way as in the analysis of Nash equilibria in Section 3, we assume that second order conditions are satisfied, that is, that $\mathcal{C}$ is concave. Conditions (10) and (11) can each be directly interpreted as the one-dimensional optimization of a DM's effort conditioned on the level of the other DM's effort *from the social planner's point of view*, rather than, as previously analyzed, from the DM's point of view. This observation will be important for later results, starting with the following.

Lemma 9. *In the case of reinforcing (conflicting) efforts, given the level of effort $\bar{e}_i$ of a DM and the best reply $\bar{e}_j^*$ of the other DM, the problem of maximizing social welfare given $\bar{e}_i$ has a unique solution $\bar{e}_j^s$, which is larger (lower) than $\bar{e}_j^*$. If e_j^* is not on the right (left) boundary of the action space, then the inequality is strict.*

Proof. See Appendix A. □

The term N_A and the symmetric N_B have a simple interpretation: they capture the spillover of DM A's effort on DM B's probability of occurrence of the bad event and the spillover of DM B's effort on DM A's probability of occurrence of the bad event. These spillovers are clearly taken into account by the planner, but not by the individual DM when he chooses his optimal effort (as in Section 2). We will denote as (e_A^C, e_B^C) the centralized optimum, and as (e_A^D, e_B^D) the decentralized Nash equilibrium.

The above result characterizes the individual and social problem when the level of effort of one DM is kept fixed; on the basis of this result, and considering now both choices of e_A and e_B simultaneously, we can derive the following results.

Proposition 10. *Let (e_A^D, e_B^D) be a Nash equilibrium under reinforcing efforts ($p_A < 0$ and $q_B < 0$). The social problem has a maximum in (e_A^C, e_B^C) such* *that* $e_A^C \geq e_A^D$ *or* $e_B^C \geq e_B^D$*, and both conditions hold if Conditions* (6) *and* (7) *also hold. The inequalities are strict except for* $e_A^D = W_A - L_A$ *or* $e_B^D = W_B - L_B$.

Proof. See Appendix A. □

An analogous result holds in the case of conflicting efforts.

Proposition 11. *Let* (e_A^D, e_B^D) *be a Nash equilibrium under conflicting efforts* ($p_A > 0$ *and* $q_B > 0$)*. The social problem has a maximum in* (e_A^C, e_B^C) *such that* $e_A^C \leq e_A^D$ *or* $e_B^C \leq e_B^D$*, and both conditions hold if Conditions* (6) *and* (7) *do not hold. The inequalities are strict except for* $e_A^D = 0$ *or* $e_B^D = 0$.

Proof. See Appendix A. □

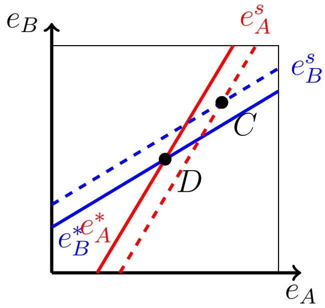
The results described above are general in the sense of not requiring unicity of Nash equilibria and social optima, but they acquire a particular interest *if* unicity is guaranteed. Indeed, in such a case they guarantee that a social planner will *necessarily* want to increase/decrease the level of effort of at least one DM, depending on the case considered.

In particular, we observe that by Lemma 8 *if* there is a unique Nash equilibrium with increasing response curves, it must be such that Condition (8) does not hold, as in Figure 7a, and the effect of centralization is non-ambiguous; whereas if there are multiple equilibria, there might be decentralized equilibria located in the opposite direction to that suggested by Propositions 10 and 11. Consider for instance Figure 7e, where C_1, the centralized optimum closest to D_1, actually envisages lower levels of effort for both DMs.

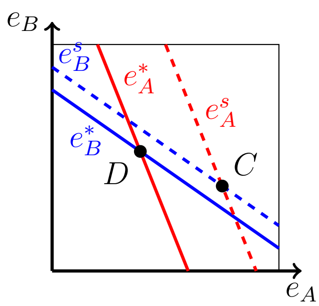
Also notice that Propositions 10 and 11 assert the presence of maxima of the social planner's problem, but do not determine whether they are global maxima. The direction of externalities may suggest that in the case of reinforcing (conflicting) efforts, local maxima with higher (lower) levels of effort will attain higher values of $\mathcal{C}$. But only in the case in which such maxima are unique (i.e., when response curves have a unique intersection) is the action of a social planner that aims for the global maximum guaranteed to go in the direction indicated by 10 and 11. In the symmetric case we can then provide an even stronger result, examined in the following proposition.

Proposition 12. *If probability and utility functions are symmetric between the two decision makers and both the centralized and the decentralized problem have a unique solution, then both* e_A *and* e_B *are larger in the centralized economy than in the decentralized economy with reinforcing efforts, and lower with conflicting efforts.*

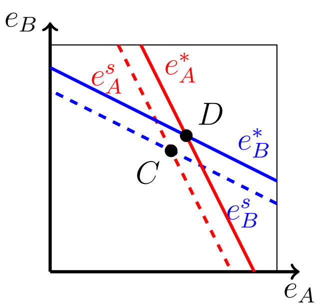
Figure 7: Comparison of centralized and decentralized equilibria



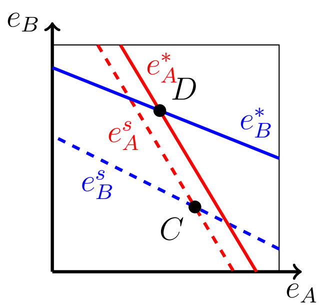
(a) Increasing response curves, reinforcing efforts



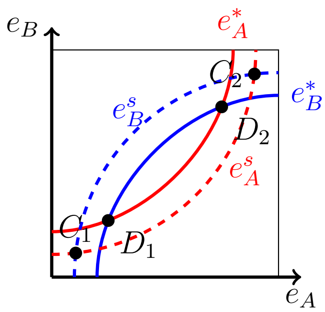
(b) Decreasing response curves, reinforcing efforts, efforts change in opposite directions



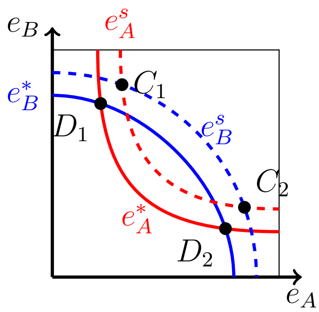
(c) Decreasing response curves, conflicting efforts, both levels of effort decrease



(d) Decreasing response curves, conflicting efforts, efforts change in opposite directions



(e) Increasing response curves, reinforcing efforts, multiple Nash equilibria



(f) Decreasing response curves, reinforcing efforts, multiple Nash equilibria

Note: Solid lines represent original response curves and D the original Nash equilibrium. Dashed lines represent socially optimal response curves and C the centralized optimum.

Proof. If the decentralized solution is unique, then it must be symmetric ($e_A^D = e_B^D$), as otherwise its symmetric (e_B^D, e_A^D) would be another solution. The same holds for the centralized solution. Propositions 10 and 11 now prove that at least one DM is changing effort level in the specified direction. Since the two DMs exert identical levels of effort in both the centralized and in the decentralized solutions, both change their effort levels in the specified direction. □

Comparing centralized and decentralized equilibria shows the complete effect of the interaction in terms of socially desirable choices. When efforts are reinforcing, there are positive spillovers from one DM's prevention to the probability of loss of the other DM. These spillovers are neglected by each DM in his decentralized choice, which implies that he exerts too low effort in prevention from a social standpoint. In the case of a unique equilibrium, this implies in turn that a social planner would ask for more effort to be exerted. When the equilibrium is symmetric, the greater effort required from a socially optimal standpoint is split equally between the two DMs. In a unique asymmetric equilibrium, the effort of at least one DM still increases, while the other might go in the opposite direction (as in Figure 7d). When efforts are conflicting the opposite occurs. Spillovers are negative, so the planner will aim for lower effort exerted in equilibrium. Again, in the case of unicity, this involves at least one DM reducing his own effort in an asymmetric setting, while the reduction is split equally between the two DMs when the equilibrium is symmetric. Seen from another point of view, if any policy intervention results in different subjects altering their behavior in opposite directions, this is due to heterogeneities in either prevention ability, or risk preferences.

5 Application to face mask use

Infective diseases in general, and COVID-19 in particular, are a very relevant application of the theory developed so far. Indeed, in this situation, multiple actors can vary the level of effort they put into preventing the spread of infection and every single actor has an effect on others' decisions.

In what follows, we disregard the relatively narrow problem of isolating individuals who are *known* to be infected, and focus instead on the more frequent problem of general measures adopted to limit the spread of contagion from *potentially infected* individuals undetected among the general population. The prototypical example of these measures is face masks. These have the advantage of having relatively well defined properties in terms of risk abatement, which depend on perseverance (keeping a mask on all the time in a social setting), correct use (for instance, covering the nose) and also on the type of mask used, as different types guarantee different levels of virus abatement. These typically correlate with higher costs, and lower comfort. However, our model can also apply to other measures such as hand washing, social distancing and avoiding gatherings. In this last case the effort consists of avoiding an enjoyable social occasion or a pleasant but crowded location.

We assume that the two decision makers considered are general members of a population, only *potentially* infected. Thus, the *a priori* probability of infection is considered to be roughly symmetrical: if A and B meet and talk in close proximity *without* masks or other protective devices, they will each have each the same probability of being infected by the other. The *effect* of protective devices can be expressed, as is common in the literature, in terms of share of pathogens blocked from reaching a potential victim (Leung et al., 2020; Lepelletier et al., 2020; Tcharkhtchi et al., 2021). For simplicity, we assume that this effect is symmetric, that is, that a mask worn by A protects both A from being infected from B and vice-versa *to the same level*. We are aware this is a simplification, given that for instance different face masks are relatively effective in stopping the inflow, or the outflow, of droplets. The model presented in the previous sections could also lend itself to modeling this aspect, but for simplicity of exposition we abstract from it in what follows. This simplification in fact helps us focus on the main phenomenon of interest, which is that individual effort simultaneously affects both one's own and others' risk.

We hence base our modeling of the problem on the following assumptions.

- Between individuals involved in a typical face–to–face interaction with no precautions taken, there is a given flow of aerosol droplets, which we take as reference for the analysis.

- Individuals can exert effort by adopting precautionary measures; a linear increase in effort translates into an exponential abatement of this flow. For instance, if a simple mask abates the flow to 40%, then wearing two such masks abates the flow *twice* by this proportion, bringing it to $40\%^2 = 16\%$: in general, levels of effort e_A and e_B result in a flow of $\alpha^{e_A+e_B}$, with $\alpha \in (0,1)$.

- The probability of infection is symmetric: $p = q$.

- If one of the two individuals is infected, then the (reference) flow of aerosol towards the other individual includes a given sample of pathogens which we normalize to 1 without loss of generality. Any measure that reduces the flow of aerosol reduces the number of pathogens proportionally.

- In accordance with the widely adopted *exponential dose–response model* (Haas, 1983; Conlan et al., 2011), we assume that the probability of infection when inhaling a given dose D is $p = 1 - e^{-rD}$, where $r \in (0,1)$ describes the "single-hit" probability – the probability of a single instance of the virus causing an infection. Given the normalization specified in the previous bullet point, we have that r is the probability of contagion in a typical interaction with no precautions taken.

- We further consider that transmission only takes place if exactly one of the two subjects is infected (omitting for simplicity the incubation period from the analysis), and that this happens with probability $i(1-i)$ for a disease with prevalence $i \in [0,1]$ in the population of interest: in particular, each individual has a probability of $\frac{i(1-i)}{2}$ of being a healthy subject meeting an infected subject.

- We limit the analysis to susceptible individuals — that is, we exclude individuals who are immune against the pathogen (e.g. vaccinated).

Given the above, we obtain the functional form

$$p(e_A, e_B) = q(e_A, e_B) = \frac{i(1-i)}{2}\left(1 - e^{-r\alpha^{e_A+e_B}}\right)$$

with $r, \alpha \in (0,1)$ the two parameters that describe the aggressiveness of the pathogen and the efficacy of prevention efforts, respectively.

We observe that entirely suppressing the flow nullifies the probability of transmission ($\lim_{e_A+e_B\to\infty} p(e_A, e_B) = 0$), which for null effort reaches a maximum value $p(0,0) = \frac{i(1-i)}{2}(1 - e^{-r})$. This maximum value is pathogen–dependent, reflecting different aspects of the epidemic at a given moment in time. In other words, it reflects the aggressivity of the pathogen but also population characteristics which may lead to a given prevalence i. For simplicity of analysis, we relabel the constant term $\frac{i(i-1)}{2}$ to c, obtaining $p(e_A, e_B) = c(1 - e^{-r\alpha^{e_A+e_B}})$.

This functional form results in partial derivatives

$$p_A(e_A, e_B) = q_B(e_A, e_B) = cr\log(\alpha)e^{-r\alpha^{e_A+e_B}}\alpha^{e_A+e_B} \tag{12}$$

and cross derivatives

$$p_{AB}(e_A, e_B) = q_{BA}(e_A, e_B) = cr\log{(\alpha)}^2 e^{-\alpha^{e_a+e_b}r}\left(\alpha^{e_a+e_b} - \alpha^{2e_a+2e_b}r\right). \tag{13}$$

The sign of p_{AB} implies that if e_B increases, the marginal effect of A's effort decreases. This might seem counterintuitive, given that an increase of e_B reduces the flow between A and B, and hence A's potential to reduce it further. However, reducing the flow also means reaching values for which its marginal effect on the probability of infection is stronger, and this second effect ultimately dominates.

The previous computations imply that

$$\frac{p_{AB}}{p_A} = \log(\alpha)(1 - r\alpha^{e_A+e_B}) \tag{14}$$

In order to explicitly parametrize preferences we assume that DM exhbits a CARA utility function, i.e. that $U(x) = -e^{\beta x}$, where $\beta > 0$ is the Arrow-Pratt coefficient of absolute risk aversion. We have shown in Section 2 that, in this case,

$$\frac{U'(W_A - L_A - e_A) - U'(W_A - e_A)}{U(W_A - L_A - e_A) - U(W_A - e_A)} = -\beta \tag{15}$$

Substituting (14) and (15) into (6) we obtain:

$$\log(\alpha)(1 - r\alpha^{e_A+e_B}) > -\beta \tag{16}$$

which in this setting of reinforcing efforts becomes a necessary and sufficient condition for response curves being *increasing*. It is worth noting that the quantity in the left-hand side of (16) does not depend on the current prevalence of the disease: it is always negative, with its absolute value increasing in e_A and e_B and bounded from above by $\log(\alpha)$. Specifically, on the basis of the value of R_0 typically attributed to COVID-19 — between 3 (Billah et al., 2020) and 7-8 (SPI-M-O, 2021) — the mean serial interval in absence of precautionary measures (estimated at 6.6 days in the phase of uncontrolled spread of the pandemic by Cereda et al., 2020) and the typical number of contacts measured in large scale studies (between 5 and 20 according to Mossong et al., 2008), we obtain that the reference probability of contagion, $1 - e^{-r}$, should be no larger than $\frac{8}{6.6 \cdot 5} \approx 0.24$. This implies that $r < 0.27$, and hence that the left side of Condition (16) takes a value close to $\log(\alpha)$ even when little or no effort is exerted. The quantity in the right-hand side of (16) is the opposite of the Arrow-Pratt index of absolute risk aversion and is negative too. Estimates of β vary significantly across studies (see Cohen and Einav, 2007) but they are usually very close to 0 (no higher than 0.01). Conversely, α is measured on a scale which goes from 0 (low–effort, efficient prevention devices) to 1 (prevention devices require large effort for even minimal prevention). Proper calibration would require definition of a nexus between for instance utility functions and available wealth, but the availability of cheap devices (face masks) which significantly decrease the flow of droplets suggests a value of α not far from 0 for COVID-19 in advanced economies. This, in turn, suggests that $\log(\alpha) \ll 0$ and Condition (16) should *never* hold.

According to Proposition 1, this analysis suggests that reaction curves in the case of face masks should definitely be decreasing: a higher level of effort on behalf of an individual will make another individual *less* willing to exert effort. Heterogeneity in effort levels across individuals, then, is perfectly consistent with the evidence, from many different countries, that levels of effort exerted in prevention during the pandemic vary significantly between individuals (Galasso et al., 2020; Fan et al., 2020; Perrotta et al., 2021).

It is clear that the evidence could be explained by other reasons, including differences in individual preferences and beliefs and differences in individual levels of knowledge and expertise on the role of protective devices. The conclusions in this work, however, provide the following complementary justification which is not based on individual differences but which is fully based on strategic behavior. Even individuals with the same preferences and beliefs may strategically adapt to each other in asymmetric equilibria where only one exerts a high level of effort. Interestingly, this may also happen as a progressive *reaction* to increased safety from contagion due to the other DM's effort (Battiston and Gamba, 2021) — it is not necessary to assume explicit strategic reasoning on behalf of individuals.

Regarding the social problem, we know from Proposition 12 that in the symmetric case, in the presence of reinforcing efforts and assuming the unicity of the centralized and decentralized solutions, a central planner wants to increase the level of effort for *both* DMs. But outside the symmetric case, the presence of decreasing response functions introduces the possibility of the two DMs changing their effort levels in *opposite directions* from the decentralized to the centralized equilibrium (recall Proposition 10 and Figure 7b). Also in this case, however, unicity will guarantee that at least one DM has to increase his effort from a social standpoint. This provides a new theoretical justification for measures taken in many countries to push people to increase their protection against possible contagion. Our analysis finds that the measures can in fact be justified by the role of positive externalities in face mask use. These are not taken into account by individuals, but need to be taken into account from the point of view of social optimality.

Lastly, our analysis allows for the possibility that central planning changes in different directions the effort levels of two individuals in the same population. This might seem counterintuive, but to put the possibility into context, consider that prevention measures deployed to curb the contagion of COVID-19 *are* in reality strongly differentiated across different segments of any country's population, on the basis of age, occupation, and location (as in the case of targeted lockdowns enacted in regions, or municipalities, where cases surge). To the extent that such focused measures reduce the likelihood of contagion between infected and susceptible individuals, they make the probability of loss more remote for individuals *not affected by such measures*, and hence they *decrease* their individual propensity to exert effort. Even accounting for compulsory measures which also apply to them, the net result might be a lower level of effort, for some individuals, than if no policy had been implemented, that is, with the pandemic completely out of control. In other words, Lemma 9 does guarantee that individuals will increase effort *conditional* on other individuals' effort levels, but they might decrease effort once taking into account that other individuals (were required to) increase their own effort levels.

6 Conclusions

When preventing a risk of incurring a bad event, an individual may, at the same time, affect the probability that the same event occurs for other people. This interaction between decisions can go in different directions: the probability of the bad event can either decrease (in what we term the *reinforcing efforts* case) or increase (*conflicting efforts*) in response to the other agent's effort.

The effects of such interaction were formalized and described first in an economy where choice is decentralized and then where the economy is centrally planned. In the decentralized economy, we examined the set of equilibria by analyzing the decision makers' reaction functions. We showed that the shape of the reaction functions depends on whether the efforts are reinforcing or conflicting, which affects marginal benefit of prevention, but also on the effects of interaction on marginal cost. The composition of these two different effects is determined by a condition comparing two elasticities. In particular, when efforts are reinforcing, reaction functions are increasing if, in the presence of an increase of effort exerted by a decision maker, the elasticity of the probability of occurrence of loss for the other decision maker is greater than the elasticity of the utility loss of the decision maker exerting the effort. Reaction functions are instead decreasing if the former is smaller than the latter. The opposite occurs when efforts are conflicting.

In all these situations, multiple equilibria may arise. In the cases where reaction curves are increasing, moving from one equilibrium to another implies that efforts exerted by both decision makers change in the same direction (they either increase or decrease together). But in cases where reaction curves are decreasing, moving from one equilibrium to another implies that efforts change in opposite directions (the effort exerted by one decision maker increases and the effort exerted by the other decreases).

Comparing these equilibria with those chosen by a central planner highlights that from a socially optimal standpoint there may be a kind of under-prevention or over-prevention in a decentralized economy. This is because individuals do not internalize into their choices the spillovers that they generate on the risks faced by other decision makers. We showed that, when the equilibrium is unique and in the case of reinforcing efforts, the central planner will require at least one DM to exert more effort than in the decentralized equilibrium, and that both DMs will do so if reaction curves are increasing or if DMs are symmetric. In the case of conflicting efforts, on the other hand, socially optimal behavior requires that at least one individual decreases his effort while all individuals are required to reduce effort if reaction curves are increasing or if DMs are symmetric.

We have shown how these general results apply to the prevention of contagion in a pandemic such as COVID-19. Efforts aimed at reducing the spread, including social distancing and mask wearing, have positive externalities, as they reduce the probability of infecting others as well as one's own probability of catching the virus (Jones et al., 2020). Our results show that, unless there is significant asymmetry between DMs, they will all increase their effort in the centralized optimum as compared to the Nash equilibrium. Our conclusions also provide a theoretical explanation in terms of strategic behavior for the evidence that levels of prevention effort during the pandemic vary significantly between individuals.

Our general results have clear implications from various standpoints. They show first that, in order to make an optimal choice in prevention effort, each individual needs to take other people's choices into account. This finding is significant from a theoretical standpoint, as, to date, it has not been taken into account in prevention models. The same finding is also important in explaining different situations emerging in society. In the case of multiple equilibria and increasing reaction curves, different equilibria are characterized by either everyone in society exerting high effort or everyone exerting low effort. Clearly, the type of equilibrium reached will depend on social habits and customs, and this also explains why people in different countries show different behaviors when facing the same risk. On the other hand, multiple equilibria in the case of decreasing reaction curves, where one individual reduces effort as the best reply to the other increasing it, may be a possible explanation of situations where significantly different levels of effort are observed within the same population.

Moreover, our analysis clearly shows that in the presence of interactions, decentralized choices may generate either under-provision or over-provision of prevention from a socially optimal standpoint. This supports the widespread adoption of public policies aimed at encouraging various forms of prevention. Our analysis provides a strong justification for such policies, implemented across different fields of the economy. Measures involving constraints existed before COVID-19, as, for example, many countries enacting legislation banning the use of alcohol or drugs before driving. The COVID-19 pandemic however is a particularly clear example of the key role of centralized decision making, for instance in the mandatory use of face masks and various lockdowns implemented across different countries. Lastly, our results are relevant for policies acting in different directions, and particularly for different forms of incentive or disincentive to prevention. In fact, it is clear that incentives, perhaps in the form of subsidies, could usefully be introduced to strengthen reinforcing efforts, and disincentives, perhaps in the form of taxation, could be useful in the case of conflicting efforts. A very simple analysis of subsidies for prevention was recently proposed by Menegatti (2021b), but while it examines the impact of some interventions, the study does not provide a foundation for the sub–optimality of decentralized equilibrium, just taking it as a fact. The present paper instead provides a basis for overcoming this limitation, and opens space for new and more in–depth future research on the issue of interaction in risk prevention. In this respect a specific aspect that may call for a fruitful investigation is the design of mechanisms of taxes/incentives that can push agents' decentralized choice towards socially optimal levels of risk prevention.

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Appendix A Proofs

Proof of Lemma 2. We prove the continuity of e_B^* (Figure 8). The continuity of e_A^* is proven symmetrically. If $\bar{e}_A$ is a point of discontinuity for e_B^*, then by definition there is an $\bar{\epsilon} > 0$ such that for every $\delta > 0$, there is a point $e_{A\delta} \in [\bar{e}_A - \delta, \bar{e}_A + \delta]$ for which $e_B^*(e_{A\delta}) \notin [e_B^*(\bar{e}_A) - \bar{\epsilon}, e_B^*(\bar{e}_A) + \bar{\epsilon}]$. If we consider $\delta_n = \frac{1}{n}$, we obtain a sequence of points $e_{A\delta_n}$ converging to $\bar{e}_A$ such that $e_B^*(e_{A\delta_n})$ is always distant at least $\bar{\epsilon}$ from $e_B^*(\bar{e}_A)$. We apply the Bolzano–Weierstrass theorem in order to extract from this sequence a subsequence $\tilde{e}_{An}$ such that $e_B^*(\tilde{e}_{An})$ has a limit $\tilde{e}_B$: clearly $\tilde{e}_B \neq e_B^*(\bar{e}_A)$. Let $\bar{v} = \mathcal{V}(\bar{e}_A, e_B^*(\bar{e}_A))$ and $\tilde{v} = \mathcal{V}(\bar{e}_A, \tilde{e}_B)$: since $\mathcal{V}$ is single–peaked in e_B (recall $\frac{\partial^2 \mathcal{V}}{\partial e_B^2} = \frac{\partial M_B}{\partial e_B} < 0$) and for $e_A = \bar{e}_A$ reaches (by definition) a maximum in $e_B^*(\bar{e}_A)$, then necessarily $\bar{v} > \tilde{v}$.

Since $\mathcal{V}$ is continuous, $\mathcal{V}(\tilde{e}_{An}, e_B^*(\tilde{e}_{An}))$ converges to $\tilde{v}$, and $\mathcal{V}(\tilde{e}_{An}, e_B^*(\bar{e}_A))$ converges to $\bar{v} > \tilde{v}$. Hence, for a large enough n, it must be that $\mathcal{V}(\tilde{e}_{An}, e_B^*(\bar{e}_A)) > \mathcal{V}(\tilde{e}_{An}, e_B^*(\tilde{e}_{An}))$. But this leads to a contradiction, because $e_B^*(\tilde{e}_{An})$ should maximize $\mathcal{V}$ given $e_A = \tilde{e}_{An}$. □

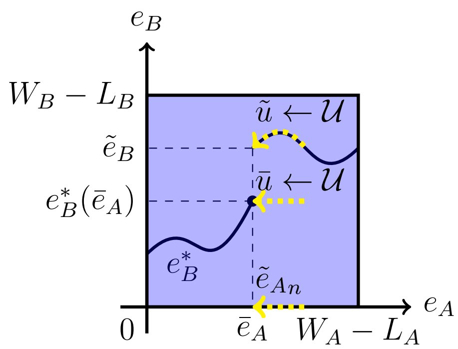


Figure 8: Proof of Lemma 2

Proof of Proposition 3. Consider the set $I_B \subset [0, W_B - L_B]$ defined as $I_B = \{e_B | e_B \leq e_B^*(e_A^*(e'_B))\}$. The set is non-empty, as it contains at least 0. If $W_B - L_B \in I_B$, this means that $W_B - L_B \leq e_B^*(e_A^*(W_B - L_B))$, but since e_B^* is bounded from above by $W_B - L_B$, then $W_B - L_B = e_B^*(e_A^*(W_B - L_B))$, and $(e_A^*(W_B - L_B), W_B - L_B)$ is a Nash equilibrium. If instead $W_B - L_B \notin I_B$, then consider $\tilde{e}_B = \sup(I_B)$ and a sequence e_{Bi} in I_B that converges to $\tilde{e}_B$. By the continuity of e_A^* and of e_B^*, we obtain $e_B^*(e_A^*(\tilde{e}_B)) = \lim_{n\to\infty} e_B^*(e_A^*(e_{Bi})) = \tilde{e}_B$. So $(e_A^*(\tilde{e}_B), \tilde{e}_B)$ is a Nash equilibrium. □

Proof of Proposition 5. Let (e_A, e_B), (e'_A, e'_B) be two Nash equilibria; without loss of generality, assume $e'_A > e_A$. If response curves are increasing, Proposition 4 guarantees that $e'_B > e_B$. Since $p_B < 0$, DM A's preferences are such that $(e_A, e'_B) \underset{A}{>} (e_A, e_B)$; moreover, since e'_A is A's unique best reply to e'_B, then $(e'_A, e'_B) \underset{A}{>} (e_A, e'_B)$; so $(e'_A, e'_B) \underset{A}{>} (e_A, e_B)$. Similarly, $(e'_A, e'_B) \underset{B}{>} (e'_A, e_B) \underset{B}{>} (e_A, e_B)$. So (e'_A, e'_B) Pareto dominates (e_A, e_B).

Vice–versa, if response curves are decreasing, Proposition 4 guarantees that $e'_B < e_B$. Since $p_B < 0$, $(e_A, e'_B) \underset{A}{>} (e_A, e_B)$, and since e'_A best replies to e'_B, $(e'_A, e'_B) \underset{A}{>} (e_A, e'_B)$; similarly, $(e_A, e_B) \underset{B}{>} (e_A, e'_B) \underset{B}{>} (e'_A, e'_B)$. So neither of the two equilibria Pareto dominates the other. □

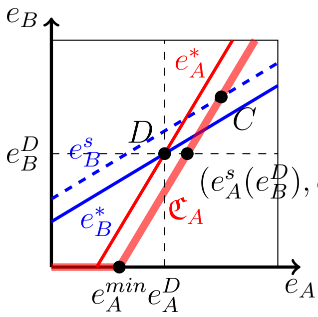
Proof of Lemma 8. Since $(\bar{e}_A, \bar{e}_B)$ is a Nash equilibrium, it is a crossing point of the response curves, and since $\frac{\partial e^*_B(\bar{e}_A)}{\partial e_A} > \frac{1}{\frac{\partial e^*_A(\bar{e}_B)}{\partial e_B}} = \frac{\partial {e^*_A}^{-1}(\bar{e}_A)}{\partial e_A}$, there is a right neighborhood of $\bar{e}_A$ where $e^*_B(e_A) > {e^*_A}^{-1}(e_A)$.

We know from Proposition 4 that the two effort levels change in the same direction from an equilibrium to another. So if there are Nash equilibria with $e_A > \bar{e}_A$, they are such that $e_B > \bar{e}_B$ too, and vice-versa. Now let us assume that there are no such Nash equilibria. There are thus no further internal crossing points of the response curves for $e_A > \bar{e}_A$ or $e_B > \bar{e}_B$, which means that the right neighborhood of $\bar{e}_A$ for which $e^*_B(e_A) > {e^*_A}^{-1}(e_A)$ is the entire $(\bar{e}_A, W_A - L_A)$ interval. Now if $e^*_B(W_A - L_A) < W_B - L_B$, then $\lim_{e_A \to W_A - L_A} {e^*_A}^{-1}(e_A) < W_B - L_B$ and by continuity there is an e'_B such that $\forall e_B \geq e'_B$, we have $e^*_A(e_B) = W_A - L_A$ (see x_3 in Figure 5c). In this case, $(W_A - L_A, e^*_B(W_A - L_A))$ is a Nash equilibrium. If instead $e^*_B(W_A - L_A) = W_B - L_B$, then $(e^*_A(W_B - L_B), W_B - L_B)$ is a Nash equilibrium (see x_3 in Figure 5d). In all cases, in the Nash equilibrium e^*_B and ${e^*_A}^{-1}$ coincide, and if Condition (8) held, it would imply (as in the reasoning above) the existence of a left neighborhood of $\bar{e}'_A$ where $e^*_B(e_A) < {e^*_A}^{-1}(e_A)$. But this is a contradiction because we know that the opposite holds in $[\bar{e}_A, \bar{e}'_A]$; so Condition (8) cannot hold. □

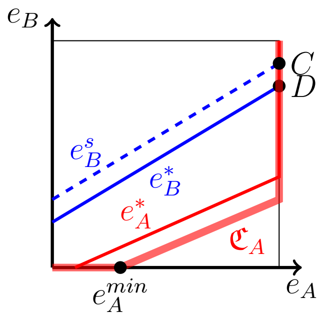
Proof of Lemma 9. Let $\bar{e}^*_A = e^*_A(\bar{e}_B)$ be the best reply of DM A to $\bar{e}_B$, and assume it is in the interior of the action space. We know it necessarily satisfies Equation (3), that is, $M_A = 0$. The socially optimal level of effort for A given $\bar{e}_B$ on the other hand satisfies Equation (10), that is, $\mathcal{C}_A = 0$. The two differ in just one term $N_A = \mathcal{C}_A - M_A = q_A(e_A, e_B)[V(W_B - L_B - e_B) - V(W_B - e_B)]$. Since V is increasing, the term between square brackets is always negative, so the sign of N_A is opposite the sign of q_A.

Let us first consider the case of reinforcing efforts ($q_A < 0$), so that $\mathcal{C}_A(\bar{e}^*_A, \bar{e}_B) = N_A(\bar{e}^*_A, \bar{e}_B) > 0$. If there exists e^s_A such that $\mathcal{C}_A(e_A, \bar{e}_B) = 0$,

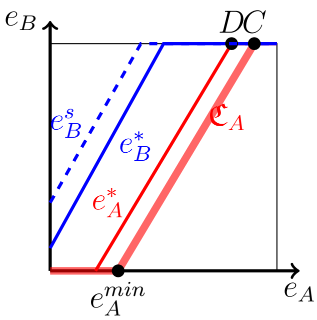
Figure 9: Illustration of proofs of Propositions 10 and 11



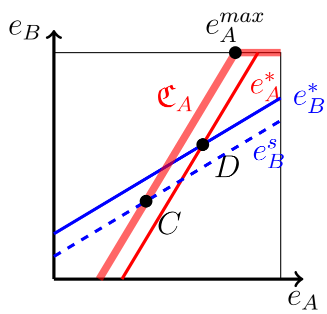
(a) Proof of Proposition 10: internal crossing point



(b) Proof of Proposition 10: case $e_B^s(W_A - L_A) < W_B - L_B$



(c) Proof of Proposition 10: case $e_B^s(W_A - L_A) = W_B - L_B$



(d) Proof of Proposition 11

Note: Solid lines represent original response curves and D the original Nash equilibrium. Dashed lines represent socially optimal response curves and C the centralized optimum.

then since we know that $\frac{\partial C_A}{\partial e_A} < 0$, necessarily $e_A^s > e_A^*$ holds. If on the other hand there is no such e_A^s, by continuity it must then be that $\mathcal{C}_A(e_A, \bar{e}_B) > 0\ \forall e_A > e_A^*$, and as a consequence $\mathcal{C}(e_A, \bar{e}_B) > \mathcal{C}(\bar{e}_A^*, \bar{e}_B)\ \forall e_A > e_A^*$. Since the problem is bounded from above, then the boundary level of effort $\bar{e}_A^s = W_A - L_A$ maximizes $\mathcal{C}(e_A, \bar{e}_B)$. So in conclusion, there is either a boundary solution, or there is instead an internal one, which is unique because of concavity.

If instead $\bar{e}_A^*$ is on the left boundary, then it must be that $M_A(e_A, \bar{e}_B) \leq 0$, and the proof is analogous. Finally, if it is on the right boundary, it must be that $M_A(e_A, \bar{e}_B) \geq 0$: hence $\mathcal{C}_A(e_A, \bar{e}_B) > 0$, and $e_A^s = e_A^*$.

Vice-versa, in the case of conflicting efforts ($q_B > 0$), we have $\mathcal{C}_A(\bar{e}_A^*, \bar{e}_B) = N_A(\bar{e}_A^*, \bar{e}_B) < 0$. Again, if there is e_A^s such that $\mathcal{C}_A(e_A, \bar{e}_B) = 0$, the second-order condition implies that $e_A^s < e_A^*$. Otherwise, $\mathcal{C}_A(\bar{e}_A^*, \bar{e}_B) < 0\ \forall e_A \in [0, \bar{e}_A^*)$ and $e_A^s = 0$ is a boundary solution. The case of e_A^* on the boundaries is analyzed symmetrically with the analysis of reinforcing efforts.

The analysis of the individual and social optimization of e_B with respect to a given $\bar{e}_A$ is symmetric to the analysis above. □

Proof of Proposition 10. We start by excluding the case $e_A^D = W_A - L_A$, which we consider later. Let $e_A^{min} = e_A^s(0)$, and consider the curve $\mathfrak{C}_A$ obtained as the union of the graph of e_A^s and the segment from $(0,0)$ to $(e_A^{min}, 0)$ (See Figure 9a). We observe that the set $\{(e_A, e_B) \in \mathfrak{C}_A | e_A = e_A^D, e_B < e_B^D\}$ is non–empty, because $\mathfrak{C}_A$ connects the point $(0,0)$ to a point $(e_A^s(e_B), e_B^D)$ which is (by Lemma 9) right of (e_A^D, e_B^D), while only intersecting each $e_B \neq 0$ exactly once, so it must pass strictly *below* (e_A^D, e_B^D). Now if $\mathfrak{C}_A$ is *above* the graph of e_B^* in $(e_A^s(e_B^D), e_B^D)$, there must be a crossing point of the two curves with $e_A \in (e_A^D, e_A^s(e_B))$, and this crossing point is a centralized optimum where A exerts an effort larger than e_A^D, which concludes the proof. If instead $\mathfrak{C}_A$ is still below the graph of e_B^* in $(e_A^s(e_B), e_B^D)$, then if there is a $e_B \in (e_B^D, W_B - L_B)$ for which $\mathfrak{C}_A$ is instead *above* the graph of e_B^*, then there must be a crossing point of the two curves with $e_B \in (e_B^D, W_B - L_B)$, and this crossing point is a centralized optimum where B exerts an effort larger than e_B^D, which again concludes the proof. Finally, if $\mathfrak{C}_A$ is below the graph of e_B^* for all $e_B \in (e_B^D, W_B - L_B)$, we distinguish two further cases: (i) $e_B^s(W_A - L_A) < W_B - L_B$, in which case $(e_A^s(W_B - L_B), W_B - L_B)$ is a centralized optimum (Figure 9b), and (ii) $e_B^s(W_A - L_A) = W_B - L_B$, in which case necessarily $(W_A - L_A, e_B^s(W_A - L_A))$ is a centralized optimum (Figure 9c).

The case $e_A^D = W_A - L_A$ is approached by reversing the role of DM A and DM B in the above proof; if $(e_A^D, e_B^D) = (W_A - L_A, W_B - L_B)$, by Lemma 8 $(e_A^C, e_B^C) = (e_A^D, e_B^D)$.

So we have proven that at least one DM increases effort in the centralized solution. Assume without loss of generality that it is DM A. If Conditions (6) and (7) hold, e_B^* is increasing, and hence by Lemma 9 $e_B^C = e_B^s(e_A^C) \geq e_B^*(e_A^C) > e_B^*(e_A^D) = e_B^D$: DM B is also increasing effort.

□

Proof of Proposition 11. The proof is symmetric to the proof of Proposition 10 In fact, it can be obtained by mirroring the action space horizontally and vertically, replacing each e_A with $W_A - L_A - e_A$ and each e_B with $W_B - L_B - e_B$ (see Figure 9d). □