On the substitution between saving and prevention

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Abstract

This work makes a joint analysis of prevention and saving decisions. First we determine the optimal levels of the two variables and we analyze substitution between them. Second we provide some comparative statics results in order to determine the effects on optimal saving and prevention of changes in exogenous present and future wealth and in possible future loss. Finally we introduce insurance into the model and we extend the separation result, derived in the literature which studies the substitution between insurance and saving, to the case where prevention is considered too.

1 Introduction

Prevention of a risk by an agent can be defined as the effort exerted by the agent in order to lower the probability of the occurrence of an event generating a loss in wealth. Given this definition, it is easy to see that, since prevention reduces the probability of a loss, it is used in order to deal with risk. The choice about it can thus be naturally related to the choice about the other instrument typically used for the same purpose, which is insurance.
This issue was originally analyzed by [Becker and Erlich (1972)], who study the substitution effects between insurace, self-insurance and prevention in an expected utility model where prevention has a simultaneous effect on the probability of the loss. In this framework, insurance and self-insurance are shown to be definitely substitutes, while insurance and prevention can be complements because of their mutually related effects on moral hazard.
On the other hand, if we examine a two-period framework, insurance can be seen as a substitute for saving, since a share of total saving, called precautionary saving, is dedicated to dealing with risk. This problem was first examined by [Moffet (1975), (1977)] under restrictive assumptions and then by [Dionne and Eeckhoudt (1984)] in a generalized framework. [Dionne and Eeckhoudt (1984)], in particular, found that a complementary or substitution effect between prevention and saving cannot be unambiguously established, in general, while perfect substitution occurs under the assumption of decreasing temporal risk aversion. Finally they show that, in the case of fair insurance premium, there is a separation between the two instruments: insurance is completely devoted to removing risk and saving is used for consumption smoothing.
The relationship between prevention and saving has not been analysed yet in this context. This is because prevention has been treated only as an instrument whose effects occur simultaneously on the probability of a loss in wealth; in fact, prevention tends to be considered only in a single-period framework. However, a prevention model in a two-period framework was recently studied by [Menegatti (2009)]. In fact, as noted by Menegatti:

"in many other situations, the effort in prevention is not contemporaneous with its effect on probability; it precedes this effect. This happens in cases such as the following three:
a) if a driver attends a safe-driving course today, this reduces the probability of a car accident in the future;
b) if a householder buys a house alarm today, this reduces the probability of a burglary in the future;
c) if a smoker gives up smoking today, this reduces the probability of disease in the future.
Clearly, in these cases the usual one-period framework cannot be used to analyse the optimal level of prevention. A two-period framework is suitable." [Menegatti (2009), p. 394]. [1]
Starting from this idea, this work aims to provide a joint analysis of prevention and saving decisions. The paper has three main goals. First we determine the optimal levels of the two choice variables and we analyse substitution or complementarity between them. Second we provide some comparative statics results in order to determine the effects on optimal saving and prevention of changes in exogenous present and future wealth and in the possible future loss. Finally we study the effect of the introduction of insurance into the model under the assumption of fair premium and we extend the [Dionne and Eeckhoudt (1984)] separation result to the case where prevention is considered.
The paper is organized as follows. Section 2 presents the model. Section 3 examines substitution between prevention and saving. Section 4 provides comparative statics results. Section 5 shows the extended separation result. Section 6 concludes.

2 The model

We assume that the agent chooses the optimal levels of saving s and prevention effort e in order to maximize her total utility $V(s, e)$ in a two-period horizon. For simplicity, we assume that wealth in period 0 (the "present") is certain, while in period 1 (the "future") the agent faces two states of the world: the "bad" one, where she incurs the loss l, and the "good" one, where no loss occurs. The agent's maximisation problem is thus

$$\underset{s,e}{max} V(s,e) = \underset{s,e}{max} \left\{ u\left(w_0 - s - e\right) + \frac{p(e)u\left(w_1 + s(1+r) - l\right) + [1-p(e)]u\left(w_1 + s(1+r)\right)}{(1+\rho)} \right\} \tag{1}$$

where $u(\,.\,)$ is the utility function, $p(e)$ is the probability that the damaging event generating the loss l occurs, w_0 and w_1 are respectively the wealth endowment of period 0 and period 1, both certain, r is the interest rate and ρ is the intertemporal discount factor.
As usual, function $u(\,.\,)$ is assumed to be strictly increasing and strictly concave, that is $u'(\,.\,) > 0$ and $u''(\,.\,) < 0$; this last assumption indicates risk aversion. We also assume $p'(e)$ to be strictly negative, so that an increase in effort causes a reduction in the probability of the loss.
Given problem (1), we get the two following first-order conditions:

$$u'(I_0) = \{p(e)u'\left(I_{1B}\right) + [1-p(e)]u'\left(I_{1G}\right)\} \frac{1+r}{1+\rho} \tag{2}$$

$$u'(I_0) = p'(e)\frac{u(I_{1B}) - u(I_{1G})}{1+\rho} \tag{3}$$

where $I_0 = w_0 - s - e$, $I_{1B} = w_1 + s(1+r) - l$, $I_{1G} = w_1 + s(1+r)$. It is worth noticing that if $I_{1B} = I_{1G}$, i.e. $l = 0$, there is no incentive to exert any effort. Conversely, the bigger the wealth gap between the two states of the world in period 1, the greater the benefit from exerting effort in the present.
The second-order conditions are

$$V_{ss} = u''(I_0) + [p(e)u''(I_{1B}) + [1-p(e)]u''(I_{1G})] \frac{(1+r)^2}{1+\rho} < 0 \tag{4}$$

$$V_{ee} = u''(I_0) + p''(e)\frac{[u(I_{1B}) - u(I_{1G})]}{1+\rho} < 0 \tag{5}$$

[1] [Menegatti (2009)] shows that, in a two-period framework, prudence, i.e. a positive third derivative of the utility function, has an increasing effect on optimal prevention. Conversely, [Eeckhoudt and Gollier (2005)] previously concluded that the opposite occurs in a one-period framework.

$$V_{ss}V_{ee} - (V_{se})^2 > 0 \tag{6}$$

where $V_{ss} = \frac{\partial^2 V(e,s)}{\partial s^2}$,$V_{ee} = \frac{\partial^2 V(e,s)}{\partial e^2}$ and $V_{se} = \frac{\partial^2 V(s,e)}{\partial e \partial s}$.
While (4) is automatically satisfied by the concavity of $u(\,.\,)$, a sufficient condition for $V_{ee} < 0$ is $p(e)$ to be convex in e. This assumption is quite natural since it generates the plausible implication that the marginal effect of prevention is decreasing for $p(e)$ approaching 1. For these reasons we explicitly assume $p''(e) > 0$.

3 Substitution between prevention and saving

As pointed out above, equations (2) and (3) together determine the set of equilibrium pairs (e, s) ensuring that agent's utility is maximised. If an equilibrium exists, though, it is not necessarily unique. In fact, looking at the expressions of the first-order conditions, it is clear that the number of equilibria depends on various factors, including the assumptions on the third derivative of the utility function with respect to saving (i.e. agent's attitude toward "prudence"), and on the third derivative of the function $p(e)$.
Although we do not know how many equilibria exist, the comparison between them allows us to get some early indications on the substitability between effort and saving. In particular, we have that:

Proposition 3.1. *Assume that there exist N equilibrium pairs (e_i, s_i) with $i = 1, .., N$. If $e_k > e_j$ with $k \neq j$, then $s_k < s_j$.*

Proof. By totally differentiating (2) we get

$$\frac{ds}{de} = \frac{-u''(I_0) - p'(e)\left[u'(I_{1B}) - u'(I_{1G})\right]\frac{1+r}{1+\rho}}{u''(I_0) + p(e)u''(I_{1B}) + (1-p(e))\,u''(I_{1G})\frac{(1+r)^2}{1+\rho}} < 0 \tag{7}$$

This means that (2) implies in equilibrium $s = f(e)$ with $f'(e) < 0$. This proves the proposition. □

Proposition 3.1 shows that, assuming that multiple equilibrium pairs (e, s) exist and comparing these different equilbria, there is a negative relationship between e and s. This means that, moving from one equilibrium to another, we have a substitution effect between prevention and saving (larger prevention implies smaller saving).
Another conclusion in the same direction can be obtained by explaining the effect on e and s of an exogenous change in the interest rate r. Indeed, since the interest rate is the return of saving, the responses of the two variables of choice to a change in it provide a further indication on the substitability between them. In order to do this, we totally differentiate (2) and (3) with respect to the endogenous variables e and s and the exogenous interest rate r, obtaining:

$$V_{ee}de + V_{es}ds = -V_{er}dr \tag{8}$$

$$V_{es}de + V_{ss}ds = -V_{sr}dr \tag{9}$$

where

$$V_{er} = \frac{\partial^2 V(e,s)}{\partial e \partial r} = p'(e)[u'(I_{1B}) - u'(I_{1G})]\frac{s}{1+\rho} < 0, \tag{10}$$

$$V_{sr} = \frac{\partial^2 V(e,s)}{\partial s \partial r} = \frac{s(1+r)}{1+\rho}\left\{p(e)\left[u''(I_{1B})\right] + \left[1-p(e)\right]u''(I_{1G})\right\} + \frac{1}{1+\rho}\left\{p(e)\left[u'(I_{1B})\right] + \left[1-p(e)\right]u'(I_{1G})\right\} \tag{11}$$

and

$$V_{es} = u''(I_0) + p'(e)\left[u'(I_{1B}) - u'(I_{1G})\right]\frac{1+r}{1+\rho} < 0 \tag{12}$$

With simple manipulations, we get

$$\frac{de}{dr} = \frac{1}{D}\left(V_{es}V_{sr} - V_{er}V_{ss}\right) \tag{13}$$

$$\frac{ds}{dr} = \frac{1}{D}\left(V_{es}V_{er} - V_{sr}V_{ee}\right) \tag{14}$$

where $D = V_{ee}V_{ss} - (V_{es})^2$, which is greater than 0 by the second-order conditions. Since the sign of V_{sr} is ambiguous, the signs of $\frac{de}{dr}$ and $\frac{ds}{dr}$ are ambiguous too. We thus try to solve the ambiguity of the sign V_{sr}.
The derivative V_{sr} measures the effect of a change in the optimal level of saving due to a change in r. In order to study this effect in more detail, we look to the simpler case where we have no uncertainty (and thus obviously no prevention). In this case the agent's problem becomes

$$\underset{s}{max} V(s) = \underset{s}{max} \left\{ u(w_0 - s) + u\left(w_1 + s(1+r)\right) \frac{1}{1+\rho} \right\} \tag{15}$$

The first-order condition of this problem is

$$u'(w_0 - s) = u'(w_1 + s(1+r)) \frac{1+r}{1+\rho} \tag{16}$$

implying

$$V_{sr} = u''(w_1 + s(1+r)) \frac{s(1+r)}{1+\rho} + u'(w_1 + s(1+r)) \frac{1}{1+\rho} \tag{17}$$

Note that the sign of the expression V_{sr} is ambiguous. This ambiguity depends on the contemporaneous presence of two effects: the substitution effect between present and future consumption related to a change of r (represented by the term $u'\left(w_1 + s(1+r)\right) \frac{1}{1+\rho}$) and the income effect due to the increase of r (represented by the term $u''\left(w_1 + s(1+r)\right) \frac{s(1+r)}{1+\rho}$). It seems plausible to assume that the first effect prevails. If this occurs, $V_{sr} > 0$.
Examining (11), it is clear that the two terms in it are similar to those in (17). For this reason we have that if the substitution effect on s of a change in r prevails on the income effect, we have $V_{sr} > 0$ also in the framework with uncertainty of (11). This, together with (13) and (14) implies in turn

Proposition 3.2. *If the substitution effect on s of a change in r prevails on the income effect due to the change, then an increase in r raises s ($\frac{ds}{dr} > 0$) and reduces e ($\frac{de}{dr} < 0$).*

Proposition 3.2 shows that when the interest rate increases, we have that, under plausible conditions, saving increases and prevention decreases. Since the interest rate is the return of saving, this confirms the conjecture that the two instruments are substitutes.

4 Comparative statics

We now examine the effect of a change in w_0, w_1 or l on the optimal choice of saving and prevention by computing $\frac{ds}{dw_0}$, $\frac{de}{dw_0}$, $\frac{ds}{dw_1}$, $\frac{de}{dw_1}$, $\frac{ds}{dl}$ and $\frac{de}{dl}$.
In general the sign of all these derivatives is ambiguous. This result is due to the interaction between the two instruments in pursuing the agent's two objectives: to smooth consumption between the two periods and to face uncertainty. Note that this result is analogous to the conclusion reached by [Dionne and Eeckhoudt (1984)], who find that changes in different parameters have, in general, ambiguous effects on the optimal levels of insurance and saving.

Given this premise, some specific conclusions can be obtained by introducing some conditions on the parameters and the functions of the model. Consider, in particular, the three following conditions:

$$p'(e)\left[u'(I_{1B}) - u'(I_{1G})\right] > \left\{p(e)u''(I_{1B}) + [1-p(e)]u''(I_{1G})\right\}(1+r) \tag{18}$$

$$p'(e)\left[u'(I_{1B}) - u'(I_{1G})\right](1+r) > p''(e)\left[u(I_{1B}) - u(I_{1G})\right] \tag{19}$$

$$p'(e)u'(I_{1B}) > p(e)u''(I_{1B})(1+r) \tag{20}$$

Given conditions (18), (19) and (20), it is possible to show that

Proposition 4.1.
- *If inequality (18) is satisfied, then $\frac{de}{dw_0} > 0$ and $\frac{de}{dw_1} > 0$. If the opposite of inequality (18) is satisfied, then $\frac{de}{dw_0} < 0$ and $\frac{de}{dw_1} < 0$.*

- *If inequality (19) is satisfied, then* $\frac{ds}{dw_0} > 0$. *If the opposite of inequality (19) is satisfied, then* $\frac{ds}{dw_0} < 0$.
- *If inequalities (18) and (19) are satisfied, then* $\frac{ds}{dw_1} < 0$. *If the opposite of inequalities (18) and (19) is satisfied, then* $\frac{ds}{dw_1} > 0$.
- *If inequality (18) and the opposite of inequality (20) are satisfied, then* $\frac{de}{dl} > 0$. *If the opposite of inequality (18) and inequality (20) are satisfied, then* $\frac{de}{dl} < 0$.
- *If inequalities (19) and (20) are satisfied, then* $\frac{ds}{dl} > 0$. *If the opposite of inequalities (19) and (20) is satisfied, then* $\frac{ds}{dl} < 0$.

Proof. See the Appendix. □

The three conditions represent the comparison between the efficiency of the two instruments with regard to different aspects. In particular, condition (18) compares the impact of a (infinitesimally small) variation of e and s on marginal intertemporal utility of saving.[2] Similarly, condition (19) compares the impact of a (infinitesimally small) variation of e and s on marginal intertemporal utility of prevention.[3] Finally, condition (20) compares the impact of a (infinitesimally small) variation of e and s on marginal intertemporal utility of saving in the bad state of the world.[4]
Given this interpretation, Proposition 4.1 shows how the sign of changes in s and e caused by changes in w_0, w_1 and l depends on the relative efficiency of the two instruments in affecting marginal intertemporal utility. In case of a change in w_0 and in w_1, all the relevant conditions refers to both states of the world. In case of a change in l, the condition on marginal utility of saving depends only on the efficiency in the bad state.

5 Extended separation result

As anticipated in Section One, [Dionne and Eeckhoudt (1984)] examine the substitution between insurance and saving in a two-period model. One of their main results shows that the introduction of a fair (or actuarial) insurance premium determines a "separation" between insurance and saving. In this case, in fact, agents choose to buy full insurance, using this instrument in order to face uncertainty. The other instrument (i.e. saving) is thus used only for the purpose of consumption smoothing between the two periods.
This separation result obviously does not hold in a context of unfair (or non-actuarial) premia. In this case, it is no longer necessarily true that insurance is the most efficient instrument to cover future losses, so saving is both an instrument to smooth consumption as well as a substitute for insurance in protection against future losses.
It can be shown that our framework too yield a separation result. In fact, introducing insurance with a fair premium into our model implies the determination of different specific goals for insurance, prevention and saving.
Allowing for the presence of insurance in our model would result in the following expression for the agent's problem:

$$\max_{s,e,X} \left\{ u\left(w_0 - s - e - \mu\frac{p(e)X}{1+r}\right) + \frac{p(e)u\Big(w_1 + s(1+r) - l + X\Big) + [1-p(e)]u\Big(w_1 + s(1+r)\Big)}{(1+\rho)} \right\} \tag{21}$$

where X is the chosen amount of insurance coverage purchased by the agent and $\mu \geq 1$ is the loading factor applied by the insurance company on the actuarial premium. From (21) we get the following

[2]Note that the left-hand side and the right-hand side of (18) are the derivatives of (2), i.e the first-order condition referred to saving, respectively with regard to e and s.

[3]Note that the left-hand side and the right-hand side of (19) are the derivatives of (3), i.e the first-order condition referred to prevention, respectively with regard to s and e.

[4]Note that the left-hand side and the right-hand side of (20) are the derivatives of (2), i.e. the first-order condition referred to saving, respectively with regard to s and e, considering just the bad state of the world.

first-order conditions:[5]

$$u'(I_0) = \frac{p'(e)\left[u(I_{1B}) - u(I_{1G})\right]}{1+\mu\frac{p'(e)X}{1+r}}\frac{1}{1+\rho} \tag{22}$$

$$u'(I_0) = \left\{p(e)u'(I_{1B}) + [1-p(e)]\,u'(I_{1G})\right\}\frac{1+r}{1+\rho} \tag{23}$$

$$u'(I_0) = \frac{1}{\mu}u'(I_{1B})\frac{1+r}{1+\rho} \tag{24}$$

where we have now $I_0 = w_0 - s - e - \mu\frac{p(e)X}{1+r}$, $I_{1B} = w_1 + s(1+r) - l + X$ and $I_{1G} = w_1 + s(1+r)$. Combining (23) and (24), we obtain the following condition

$$\frac{u'(I_{1G})}{u'(I_{1B})} = \frac{1-p(e)\mu}{\mu\,[1-p(e)]} \tag{25}$$

From (25) it is cleat that, when the insurance premium is fair, i.e. when $\mu = 1$, $u'(I_{1G})$ must equal $u'(I_{1B})$, which implies

$$X = l \tag{26}$$

By using (22), (23) and (26) we get [6]

$$p'(e) = -\frac{1+r}{l} \tag{27}$$

and

$$u'\left(w_0 - s - e - \frac{p(e)l}{1+r}\right) = u'\Big(w_1 + s(1+r)\Big)\frac{1+r}{1+\rho} \tag{28}$$

Equations (26),(27) and (28) show the specific goal for each instrument.
First, equation (26) shows that the agent chooses a level of insurance equal to her loss, implying that she chooses full insurance. This means that insurance is used by the agent to completely remove uncertainty.
Given full insurance, the choice of the level of prevention does not influence future wealth, which is now certain. This choice, however, affects present wealth in two opposite directions since it determines the level of the fair premium and the cost due to effort exerted. In particular, a larger e implies both a larger effort exerted (reducing wealth in period 0) and a smaller fair premium $\frac{p(e)l}{1+r}$, (increasing wealth in period 0). The choice of e in equation (27) ensures that the balance between these two effects is such that the quantity $-\frac{p(e)l}{1+r} - e$ is minimized, and thus that wealth in period 0 is maximized (with regard to e). In other words the choice of e is such that the insurance premium paid is reduced until the marginal benefit from this reduction is larger than the marginal effort necessary to obtain it.
Finally, given full insurance and optimal prevention determined by (27), the agent has to choose the optimal allocation of consumption between the two periods. This is ensured by the level of saving determined in equation (28), where (given e from (27)) the expected marginal utilities in the two periods are equalised by the choice of s. Saving is thus dedicated to the purpose of consumption smoothing.
The extended separation result just described also provides a clear indication with regard to comparative statics in the case of fair insurance premium. In particular, in this case, it is easy to see that optimal insurance X is determined by the level of the loss l. Obviously, since we desire full insurance, an increase in loss implies an increase in the insurance level.
Optimal prevention e is affected by l and r. In particular, since $p'(e) < 0$ and $p''(e) > 0$ it is clear that we have $\frac{de}{dl} > 0$ and $\frac{de}{dr} < 0$. These results indicate that a larger l (r) implies a larger (smaller) insurance premium, inducing in turn a larger (smaller) prevention.
Lastly optimal saving s is affected by w_0, w_1, ρ and r. In particular we have $\frac{ds}{dw_0} > 0$, $\frac{ds}{dw_1} < 0$, $\frac{ds}{d\rho} < 0$ and $\frac{ds}{dr}$ ambiguous. In fact, considering equation (28), since $u'' < 0$, an increase in w_0 implies a decrease in the left-hand side of the equation. Since the right-hand side is not changed and since $\frac{de}{dw_0} = 0$, equation (28) is now satisfied only if saving is larger. Similarly, if w_1 increased then the

[5] Note that the first of these conditions requires $p'(e)$ to belong to $\left(-\frac{1+r}{\mu X},\, 0\right)$ since the denominator of the right-hand side cannot be negative.

[6] This is clear since, by substituting (26) in (22), we get $u'(I_0)[1+\mu\frac{p'(e)l}{1+r}] = 0$.

right-hand side of (28) decreases. Since the left-hand side is not changed and since $\frac{de}{dw_1} = 0$, equation (28) is now satisfied if saving is smaller. Again, if ρ is larger the right-hand side of (28) is smaller, implying that, since the left-hand side is not changed and since $\frac{de}{d\rho} = 0$, s has to be smaller for equation (28) to be satisfied. Finally, with regard to the effect on s of a change in r, from condition (28) it follows that

$$\frac{ds}{dr} = \frac{-u''\left(w_0 - s - e - \frac{p(e)l}{1+r}\right)\frac{p(e)l}{(1+r)^2} + u''\Big(w_1 + s(1+r)\Big)s\frac{1+r}{1+\rho} + u'\Big(w_1 + s(1+r)\Big)\frac{1}{1+\rho}}{-u''\left(w_0 - s - e - \frac{p(e)l}{1+r}\right) - u''\Big(w_1 + s(1+r)\Big)\frac{(1+r)^2}{1+\rho}} \tag{29}$$

meaning that the sign of $\frac{ds}{dr}$ is ambiguous.
It is important to emphasise that the four derivatives computed ($\frac{ds}{dw_0}$, $\frac{ds}{dw_1}$, $\frac{ds}{d\rho}$ and $\frac{ds}{dr}$) describe effects which are the same of a simple two-period problem in certainty framework. The first two describe the effect of present and future wealth on smoothing, the third describes the effect of impatience on the saving level, while the last spells out the usual effect of the return from savings (described by equations (15),(16) and (17) in Section Three).[7]
As in the case studied by [Dionne and Eeckhoudt (1984)], when considering an unfair premium, i.e when $\mu > 1$, the full insurance condition no longer holds and the separation between the instruments disappears. In this case, all three instrument act together to face disutility due to uncertainty, to reduce insurance premium and to optimally allocate wealth between the two periods. For this reason, for $\mu > 1$, we find that the sign of the effects of parameter changes is ambiguous.
We emphasize that these last results are coherent with the findings of Dionne and Eeckhoudt, who obtain a clear direction for some effects only by imposing specific conditions in the case with saving and insurance (and no prevention). Similar conclusions on comparative statics are obtained in Section Three in the case with saving and prevention (and no insurance). The results obtained in the presence of the three instruments together and for $\mu > 1$ are however much more complex and no simple condition ensuring clear indications on them can be derived. For this reason we choose to omit the findings for $\mu > 1$.

6 Conclusion

This work examines the optimal choice of savings and prevention in a two-period model with uncertain future wealth. Three main aspects were analysed.
First, with regard to the relationship between the optimal levels of the two instruments, we found that there is a kind of substitution between prevention and saving in equilibrium. In fact, comparing different equilibria, if they exist, we found that they are characterised by a decreasing relationship between saving and prevention (If saving is larger in one equilibrium than in another, then prevention is smaller). Furthermore, in the same context, we showed that a change in the interest rate (which is the return of saving) has, under usual conditions, an opposite effect on the two instruments, increasing saving and decreasing prevention.
Second, we examined comparative statics, studying the effects on prevention and saving of changes in the endowment wealth w_0 and w_1 and in the loss l. We showed that, in general, all these effects are ambiguous since, as the two instruments are substitutes, we cannot determine the signs of the changes in their optimal levels without introducing specific assumptions. However, clearly determined results can be derived by introducing conditions on the relative efficiency of the two choice variables on intertemporal marginal utility. We showed that different combinations of these conditions allow to indicate the direction of all the effects analysed.
Finally we extended the separation result derived, given a fair insurance premium, by [Dionne and Eeckhoudt (1984)] in the case of two instruments (insurance and saving) to the case of three instruments (insurance, saving and prevention). We showed that, in this case, full insurance is used in order to remove uncertainty, prevention is used in order to reduce the insurance premium and saving is used for smoothing. Under the same assumptions we also provide some comparative statics results confirming this interpretation.

[7] Equation (29) incudes the term $-u''(I_0)\frac{p(e)l}{(1+r)^2}$, absent in (17), which is due to the effect of r on wealth in period 0.

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7 Appendix

In order to provide comparative statics for prevention and saving and to prove the results in Proposition 4.1, we totally differentiate (2) and (3) with respect to the exogenous parameters w_0, w_1 and l. Letting $V_{sk} = \frac{\partial^2 V}{\partial s \partial k}$ and $V_{ek} = \frac{\partial^2 V}{\partial e \partial k}$ we get

$$V_{ee}de + V_{es}ds = -V_{ek}dk \tag{30}$$

$$V_{es}de + V_{ss}ds = -V_{sk}dk \tag{31}$$

for $k = w_0, w_1, l$, where $V_{ee} = u''(I_0) + p''(e)\frac{[u(I_{1B})-u(I_{1G})]}{1+\rho} < 0$, $V_{ss} = u''(I_0) + [p(e)u''(I_{1B}) + [1 - p(e)]u''(I_{1G})\frac{(1+r)^2}{1+\rho} < 0$, $V_{es} = u''(I_0) + p'(e)\left[u'(I_{1B}) - u'(I_{1G})\right]\frac{1+r}{1+\rho} < 0$.
By combining (30) and (31) we get

$$\frac{de}{dk} = \frac{1}{D}\left[V_{sk}V_{es} - V_{ek}V_{ss}\right]$$

$$\frac{ds}{dk} = \frac{1}{D}\left[V_{ek}V_{es} - V_{sk}V_{ee}\right]$$

for $k = w_0, w_1, l$ and where $D = V_{ee}V_{ss} - (V_{es})^2 > 0$.Given this general premise, we now consider the three specific cases where k is w_0, w_1 or l. Note that we present proofs for derivatives to be positive. The proofs for negative derivatives follows the same steps.

- $k = w_0$
  In this case we get that
  $$V_{sw_0} = V_{ew_0} = -u''(I_0) > 0$$
  We can then collect V_{sw_0} in equations (30) and (31), obtaining
  $$\frac{de}{dw_0} = \frac{1}{D}V_{sw_0}\left(V_{es} - V_{ss}\right)$$
  $$\frac{ds}{dw_0} = \frac{1}{D}V_{sw_0}\left(V_{es} - V_{ee}\right)$$
  Since V_{sw_0} is positive, the sings of $\frac{de}{dw_0}$ and $\frac{ds}{dw_0}$ are determined respectively by the sign of $V_{es} - V_{ss}$ and $V_{es} - V_{ee}$. These differences are equal to
  $$V_{es} - V_{ss} = p'(e)\left[u'(I_{1B}) - u'(I_{1G})\right]\frac{1+r}{1+\rho} - \left[p(e)u''(I_{1B}) + [1-p(e)]\,u''(I_{1G})\right]\frac{(1+r)^2}{1+\rho}$$

$$V_{es} - V_{ee} = p'(e)\left[u'(I_{1B}) - u'(I_{1G})\right]\frac{1+r}{1+\rho} - p''(e)\left[u(I_{1B}) - u(I_{1G})\right]\frac{1}{1+\rho}$$

By these two equations it can be easily seen, after simple computations, that sufficient conditions for $V_{es} - V_{ss}$ and $V_{es} - V_{ee}$ to be positive are respectively (18) and (19) □

- $k = w_1$
  In this case, we have

$$\frac{de}{dw_1} = \frac{1}{D}\left[V_{sw_1}V_{es} - V_{ew_1}V_{ss}\right] =$$

$$\frac{1}{D}\Big[\left[p(e)u''(I_{1B}) + [1-p(e)]u''(I_{1G})\right]\frac{1+r}{1+\rho} \times p'(e)\left[u'(I_{1B} - u'(I_{1G})\right]\frac{1+r}{1+\rho} +$$

$$-\left[p(e)u''(I_{1B}) + [1-p(e)]u''(I_{1G})\right]\frac{(1+r)^2}{1+\rho} \times p'(e)[u'(I_{1B}) - u'(I_{1G})]\frac{1}{1+\rho} +$$

$$+u''(I_0)\left[p(e)u''(I_{1B}) + [1-p(e)]u''(I_{1G})\right]\frac{1+r}{1+\rho} - u''(I_0)p'(e)\left[u'(I_{1B}) - u'(I_{1G})\right]\frac{1}{1+\rho}\Big]$$

Simple computations show that a sufficient condition for the last espression to be positive is

$$p'(e)\left[u'(I_{1B}) - u'(I_{1G})\right] > \left[p(e)u''(I_{1B}) + [1-p(e)]u''(I_{1G})\right](1+r)$$

which is inequality (18). □
With regard to the sign of $\frac{ds}{dw_1} = \frac{1}{D}\left[V_{ew_1}V_{es} - V_{sw_1}V_{ee}\right]$ note that, as stated before, condition (19) ensures that $V_{es} - V_{ee} > 0$, or, equivalently, $V_{es} > V_{ee}$. Given that both V_{es} and V_{ee} are negative quantities, the opposite of (19) guarantees that $|V_{es}| > |V_{ee}|$. Now computing V_{sw_1} and V_{ew_1} we get

$$V_{sw_1} = \{p(e)u''(I_{1B}) + [1-p(e)]u''(I_{1G})\}\frac{(1+r)}{1+\rho} < 0$$

$$V_{ew_1} = p'(e)[u'(I_{1B}) - u'(I_{1G})]\frac{1}{1+\rho} < 0$$

Then, holding the opposite of (19), and since both V_{ew_1} and V_{sw_1} are negative, it is sufficient to find a condition for which $|V_{ew_1}|$ is greater than $|V_{sw_1}|$ to claim that $\frac{ds}{dw_1} > 0$.
Obviously $|V_{ew_1}| > |V_{sw_1}|$ if $V_{sw_1} > V_{ew_1}$. After easy manipulations, we get that this last inequality is satisfied if:

$$\left[p(e)u''(I_{1B}) + [1-p(e)]u''(I_{1G})\right](1+r) > p'(e)\left[u'(I_{1B}) - u'(I_{1G})\right]$$

which is the opposite of inequality (18). □

- $k = l$
  First we determine the sign of V_{el} and V_{sl}:

$$V_{el} = -p'(e)[u'(I_{1B})]\frac{1}{1+\rho} > 0$$

$$V_{sl} = -p(e)\left[u''(I_{1B})\right]\frac{1+r}{1+\rho} > 0$$

Let now suppose that (18) holds: as proven before, this guarantees that $|V_{ss}| > |V_{es}|$, or, equivalently, that $V_{es} > V_{ss}$, given that both V_{ss} and V_{es} are negative. Given that $\frac{de}{dl} = \frac{1}{D}\left[V_{sl}V_{es} - V_{el}V_{ss}\right]$, in order to claim that $\frac{de}{dl} > 0$, we need $V_{el} > V_{sl}$. This occurs if

$$p(e)u''(I_{1B})(1+r) > p'(e)u'(I_{1B})$$

which is the opposite of inequality (20). □
To ensure the positivity of $\frac{ds}{dl} = \frac{1}{D}\left[V_{el}V_{es} - V_{sl}V_{ee}\right]$, we finally need both $V_{sl} > V_{el}$ and $|V_{ee}| > |V_{es}|$. Given the previous results, the former is implied by condition (20), while the latter is implied by condition (19). □